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Proceedings of 19th Annual Meeting in Mathematics (AMM2014)
Thammasat University, Rangsit Center, Pathum Thani, Thailand, March 20-22, 2014
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Soft Homomorphisms of k-Soft Hypergroups

W. Phanthawimol[1] **and P. Yoosomran**[2,*]

[1]Department of Mathematics, Faculty of Sciences
Ramkhamhaeng University, Bangkok 10240, THAILAND
golfma35@yahoo.com
[2]Department of Mathematics, Faculty of Sciences
Ramkhamhaeng University, Bangkok 10240, THAILAND
fscipsy@ku.ac.th

Abstract : Soft set theory was introduced by Molodtsov in 1999 as a general mathematical tool for dealing with problems that contain uncertainity. In this paper, we provide a definition of k-soft hypergroups and study some properties of homomorphisms of hypergroups $(\mathbb{Z}, \circ_{m\mathbb{Z}})$ and $(\mathbb{Z}_n, \circ_{m\mathbb{Z}_n})$ that induce soft homomorphisms on k-soft hypergroup and k_n-soft hypergroup.

Keywords : hyperstructure; soft set; soft hypergroup; soft homomorphism.
2010 Mathematics Subject Classification : 20N20

1 Introduction

The set of integers, the set of rational numbers and the set of real numbers are denoted by $\mathbb{Z}$, $\mathbb{Q}$ and $\mathbb{R}$, respectively.

A hyperoperation on a non-empty set H is a function $\circ : H \times H \to P(H) \smallsetminus \{\varnothing\}$ where $P(H)$ is the power set of H. The value of $(x,y) \in H \times H$ under $\circ$ is denoted by $x \circ y$ which is called the *hyperproduct* of x and y. The system $(H, \circ)$ is called a *hypergroupoid*. For $A, B \subseteq H$ and $x \in H$, let

$$A \circ B = \bigcup_{\substack{a \in A \\ b \in B}} a \circ b, \ A \circ x = A \circ \{x\} \text{ and } x \circ A = \{x\} \circ A.$$

The hypergroupoid $(H, \circ)$ is called a *semihypergroup* if

$$(x \circ y) \circ z = x \circ (y \circ z) \quad \text{for all } x, y, z \in H.$$

A *hypergroup* is a semihypergroup $(H, \circ)$ satisfying the condition

$$H \circ x = x \circ H = H \quad \text{for all } x \in H.$$

Then hypergroups are a generalization of groups.

Let $(H, \circ)$ be a hypergroup and K be a non-empty subset of H, then K is called a *subhypergroup* if $(K, \circ)$ is a hypergroup.

*Corresponding author e-mail : dreeard@yahoo.com

Let $(H_1,\circ)$ and $(H_2,*)$ be two hypergroupoids. The map $f: H_1 \to H_2$ is called a *good homomorphism* if for all $x,y \in H_1$ the following relation holds: $f(x\circ y) = f(x)*f(y)$ and is called an *(inclusion) homomorphism* if $f(x\circ y) \subseteq f(x)*f(y)$.

We denoted $\mathrm{Hom}((H_1,\circ),(H_2,*))$, $\mathrm{GHom}((H_1,\circ),(H_2,*))$, the set of all (inclusion) homomorphism and the set of all good homomorphism from $(H_1,\circ)$ into $(H_2,*)$, respectively. Let $\mathrm{Hom}(H,\circ)$, $\mathrm{GHom}(H,\circ)$ stand for $\mathrm{Hom}((H,\circ),(H,\circ))$ and $\mathrm{GHom}((H,\circ),(H,\circ))$ respectively. For $f \in \mathrm{Hom}((H_1,\circ),(H_2,*))$, f is called an epimorphism if $f(H_1) = H_2$. We denote $\mathrm{Epi}((H_1,\circ),(H_2,*))$ and $\mathrm{GEpi}((H_1,\circ),(H_2,*))$ for the set of all epimorphisms and the set of all good epimorphisms from $(H_1,\circ)$ into $(H_2,*)$, respectively. Let $\mathrm{Epi}(H,\circ)$, $\mathrm{GEpi}(H,\circ)$ stand for $\mathrm{Epi}((H,\circ),(H,\circ))$ and $\mathrm{GEpi}((H,\circ),(H,\circ))$ respectively.

Example 1.1. *[2] Let G be a group and N a normal subgroup of G. If $\circ_N$ is the hyperoperation defined on G by*

$$x \circ_N y = xyN \quad \text{for all } x,y \in G,$$

then $(G,\circ_N)$ is a hypergroup.

Example 1.2. *[2] Let G be a group. For $x,y \in G$, define*

$$x \circ y = < x,y >, \text{ the subgroup of } G \text{ generated by } x \text{ and } y,$$

then $(G,\circ)$ is a hypergroup. Note that if $(A,+)$ is an abelian group, then $x\circ y = \mathbb{Z}x + \mathbb{Z}y$ for all $x,y \in A$.

Let U be an initial universe set and E be a set of parameters. The power set of U is denoted by $P(U)$ and A is a subset of E.

A pair (F,A) is called a *soft set* over U, where F is a mapping given by

$$F: A \to P(U).$$

The set $Supp(F,A) = \{x \in A | F(x) \neq \varnothing\}$ is called the *support* of the soft set (F,A). If $Supp(F,A) \neq \varnothing$, then a soft set (F,A) is called *non-null*.

Let (F,A) and (G,B) be two soft sets over U and U' respectively, $f: U \to U'$, $g: A \to B$ be two functions. Then we say that pair (f,g) is a *soft function* from (F,A) to (G,B) denoted by $(f,g): (F,A) \to (G,B)$ if it satisfies

$$f(F(x)) = G(g(x))$$

for all $x \in A$. If f and g are injective (resp. surjective, bijective), then (f,g) is said to be injective (resp. surjective, bijective).

Example 1.3. *[1] Let us consider a soft set (F,E) which describes the "attractiveness of houses" that Mr. X is considering for purchase.*

Suppose that there are six houses in the universe $U = \{h_l,h_2,h_3,h_4,h_5,h_6\}$ under consideration, and that $E = \{e_l,e_2,e_3,e_4,e_5\}$ is a set of decision parameters. The $e_i(i = 1,2,3,4,5)$ stand for the parameters "expensive", "beautiful", "wooden", "cheap" and "in green surroundings" respectively.

Consider the mapping F given by "houses(*)", where $(*)$ is to be filled in by one of the parameters $e_i \in E$. For instance, $F(e_1)$ means "houses(expensive)", and its functional value is the set $\{h \in U : h \text{ is an expensive house}\}$.

Suppose that $F(e_1)=\{h_2,h_4\}$, $F(e_2)=\{h_1,h_3\}$, $F(e_3)=\varnothing$, $F(e_4)=\{h_1,h_3,h_5\}$ and $F(e_5)=\{h_1\}$. Then we can view the soft set (F,E) as consisting of the following collection of approximations:

$$(F,E)=\{(\textit{expensive houses},\{h_2,h_4\}),(\textit{beautiful houses},\{h_1,h_3\}),(\textit{wooden houses},\varnothing),$$
$$(\textit{cheap houses},\{h_1,h_3,h_5\}),(\textit{in the green surroundings},\{h_1\})\}.$$

Each approximation has two parts: a predicate and an approximate value set.

2 Preliminaries

2.1 Hypergroups

From Example 1.1, let $(\mathbb{Z},+),(\mathbb{Z}_n,+)$ be groups and $m\mathbb{Z},m\mathbb{Z}_n$ are subgroups of $(\mathbb{Z},+),(\mathbb{Z}_n,+)$, respectively. Then $(\mathbb{Z},\circ_{m\mathbb{Z}})$ and $(\mathbb{Z}_n,\circ_{m\mathbb{Z}_n})$ are the hypergroups where

$$x\circ_{m\mathbb{Z}}y=x+y+m\mathbb{Z}\qquad\forall x,y\in\mathbb{Z}$$
$$\overline{x}\circ_{m\mathbb{Z}_n}\overline{y}=\overline{x}+\overline{y}+m\mathbb{Z}_n\quad\forall x,y\in\mathbb{Z}.$$

Theorem 2.1. *[4] For $f:\mathbb{Z}\to\mathbb{Z}$, the following statements are equivalent:*

(i) $f\in\mathrm{Hom}(\mathbb{Z},\circ_{m\mathbb{Z}})$.

(ii) $f(x+m\mathbb{Z})\subseteq xf(1)+m\mathbb{Z}$ *for all* $x\in\mathbb{Z}$.

(iii) There exists an integer a such that

$$f(x+m\mathbb{Z})\subseteq xa+m\mathbb{Z}.$$

Theorem 2.2. *[4] For $f:\mathbb{Z}\to\mathbb{Z}$, $f\in\mathrm{Epi}(\mathbb{Z},\circ_{m\mathbb{Z}})$ if and only if*

(i) $f(x+m\mathbb{Z})=xf(1)+m\mathbb{Z}$ *for all* $x\in\mathbb{Z}$ *and*

(ii) $f(1)$ *and m are relatively prime.*

Theorem 2.3. *[4] For $f:\mathbb{Z}_n\to\mathbb{Z}_n$, the following statements are equivalent:*

(i) $f\in\mathrm{Hom}(\mathbb{Z}_n,\circ_{m\mathbb{Z}_n})$.

(ii) $f(\overline{x}+m\mathbb{Z}_n)\subseteq xf(\overline{1})+m\mathbb{Z}_n$ *for all* $x\in\mathbb{Z}$.

(iii) There exists an integer a such that

$$f(\overline{x}+m\mathbb{Z}_n)\subseteq x\overline{a}+m\mathbb{Z}_n.$$

Theorem 2.4. *[4] For $f:\mathbb{Z}_n\to\mathbb{Z}_n$, $f\in\mathrm{Epi}(\mathbb{Z}_n,\circ_{m\mathbb{Z}_n})$ if and only if*

(i) $f(\overline{x}+m\mathbb{Z}_n)=xf(\overline{1})+m\mathbb{Z}_n$ *for all* $x\in\mathbb{Z}$ *and*

(ii) If $f(\overline{1})=\overline{a}$ *for* $a\in\mathbb{Z}$*, then a and* (m,n) *are relatively prime.*

2.2 Soft Hypergroup

Let (F,A) be a non-null soft set over H. Then (F,A) is called a *soft hypergroup* over H if $F(x)$ is a subhypergroup of H for all $x \in Supp(F,A)$.

Let (F,A) and (G,B) be two non-empty soft hypergroupoids over H_1 and H_2, respectively. Let (f,g) be a soft function from (F,A) to (G,B). If f is an inclusion (resp. good) homomorphism from H_1 to H_2, then (f,g) is said to be a soft inclusion (resp. good) homomorphism of hypergroupoids.

For soft hypergroups (F,A) and (G,B) over H_1 and H_2 respectively. Denote the set of all soft inclusion homomorphisms and the set of all soft good homomorphisms from (F,A) to (G,B) by $\text{SfHom}((F,A)_{(H_1,\circ)},(G,B)_{(H_2,*)})$ and $\text{SfGHom}((F,A)_{(H_1,\circ)},(G,B)_{(H_2,*)})$, respectively. Let $\text{SfHom}((F,A),(G,B))_{(H,\circ)}$ and $\text{SfGHom}((F,A),(G,B))_{(H,\circ)}$ stand for $\text{SfHom}((F,H)_{(H,\circ)},(G,H)_{(H,\circ)})$ and $\text{SfGHom}((F,H)_{(H,\circ)},(G,H)_{(H,\circ)})$ respectively.

2.3 k-Soft Hypergroup and k_n-Soft Hypergroup

For $m \in \mathbb{Z}^+$. Let $F : \mathbb{Z} \to P(\mathbb{Z})$ be a mapping given by

$$F(x) = \begin{cases} x\mathbb{Z} & x|m \\ \mathbb{Z} & x \nmid m. \end{cases}$$

Then $(F,\mathbb{Z})$ is a soft set over $\mathbb{Z}$. Since $F(x) \neq \varnothing$ for all $x \in \mathbb{Z}$, then $Supp(F,\mathbb{Z}) \neq \varnothing$. Hence $(F,\mathbb{Z})$ is a non-null.

Proposition 3.1 and 3.2 are shown that $(k\mathbb{Z},\circ_{m\mathbb{Z}})$, $(k\mathbb{Z}_n,\circ_{m\mathbb{Z}_n})$ are subhypergroups of $(\mathbb{Z},\circ_{m\mathbb{Z}})$, $(\mathbb{Z}_n,\circ_{m\mathbb{Z}_n})$ respectively. For a hypergroup $(\mathbb{Z},\circ_{m\mathbb{Z}})$. If $k|m$, then $(F_k,\mathbb{Z})$ is called a *k-soft hypergroup* over $\mathbb{Z}$ where $F_k : \mathbb{Z} \to P(\mathbb{Z})$ defined by

$$F_k(x) = \begin{cases} k\mathbb{Z} & x|m \\ \mathbb{Z} & x \nmid m. \end{cases}$$

For a hypergroup $(\mathbb{Z}_n,\circ_{m\mathbb{Z}_n})$. If $(k,n)|(m,n)$, then $(F_k,\mathbb{Z}_n)$ is called a *k_n-soft hypergroup* over $\mathbb{Z}_n$ where $F_k : \mathbb{Z}_n \to P(\mathbb{Z}_n)$ defined by

$$F_k(x) = \begin{cases} k\mathbb{Z}_n & (x,n)|(m,n) \\ \mathbb{Z}_n & (x,n) \nmid (m,n). \end{cases}$$

3 Main Results

Throughout this Section, let $(\mathbb{Z}, \circ_{m\mathbb{Z}})$ and $(\mathbb{Z}_n, \circ_{m\mathbb{Z}_n})$ be hypergroups, $(F_k, \mathbb{Z})$ and $(G_k, \mathbb{Z})$ be k-soft hypergroups over $\mathbb{Z}$ and $(F_k, \mathbb{Z}_n)$ and $(G_k, \mathbb{Z}_n)$ be k_n-soft hypergroups over $\mathbb{Z}_n$.

Proposition 3.1. *For $m \in \mathbb{Z}^+$, if $k|m$ then $(k\mathbb{Z}, \circ_{m\mathbb{Z}})$ is a subhypergroup of $(\mathbb{Z}, \circ_{m\mathbb{Z}})$.*

Proof. Since $k|m$ then $m\mathbb{Z} \leq k\mathbb{Z}$. Thus for every $x \in k\mathbb{Z}$,

$$\begin{aligned} k\mathbb{Z} \circ_{m\mathbb{Z}} x &= \bigcup_{t \in k\mathbb{Z}} (t \circ_{m\mathbb{Z}} x) \\ &= \bigcup_{t \in k\mathbb{Z}} (t + x + m\mathbb{Z}) \\ &= k\mathbb{Z} + x + m\mathbb{Z} = k\mathbb{Z}. \end{aligned}$$

Similarly, $x \circ_{m\mathbb{Z}} k\mathbb{Z} = k\mathbb{Z}$.
Hence $(k\mathbb{Z}, \circ_{m\mathbb{Z}})$ is a subhypergroup of $(\mathbb{Z}, \circ_{m\mathbb{Z}})$. □

Proposition 3.2. *For $m, n \in \mathbb{Z}^+$, if $(k,n)|(m,n)$ then $(k\mathbb{Z}_n, \circ_{m\mathbb{Z}_n})$ is a subhypergroup of $(\mathbb{Z}_n, \circ_{m\mathbb{Z}_n})$.*

Proof. Since $(k,n)|(m,n)$ then $m\mathbb{Z}_n \leq k\mathbb{Z}_n$. Thus for every $\overline{x} \in k\mathbb{Z}_n$

$$\begin{aligned} k\mathbb{Z} \circ_{m\mathbb{Z}_n} \overline{x} &= \bigcup_{\overline{t} \in k\mathbb{Z}} (\overline{t} \circ_{m\mathbb{Z}_n} \overline{x}) \\ &= \bigcup_{\overline{t} \in k\mathbb{Z}} (\overline{t} + \overline{x} + m\mathbb{Z}_n) \\ &= k\mathbb{Z}_n + \overline{x} + m\mathbb{Z}_n = k\mathbb{Z}_n. \end{aligned}$$

Similarly, $\overline{x} \circ_{m\mathbb{Z}_n} k\mathbb{Z}_n = k\mathbb{Z}_n$.
Hence $(k\mathbb{Z}_n, \circ_{m\mathbb{Z}_n})$ is a subhypergroup of $(\mathbb{Z}_n, \circ_{m\mathbb{Z}_n})$. □

Theorem 3.3. *For $m, k \in \mathbb{Z}^+$ and $k|m$. If $f \in \mathrm{Hom}(\mathbb{Z}, \circ_{m\mathbb{Z}})$, then $f(x + m\mathbb{Z}) \subseteq k\mathbb{Z}$ for all $x \in k\mathbb{Z}$.*

Proof. Assume $f \in \mathrm{Hom}(\mathbb{Z}, \circ_{m\mathbb{Z}})$ and $k|m$. Then $m = ks$ for some $s \in \mathbb{Z}$. Let $x \in k\mathbb{Z}$. Then $x = kt$ for some $t \in \mathbb{Z}$. Since $f(a + m\mathbb{Z}) \subseteq af(1) + m\mathbb{Z}$ for all $a \in \mathbb{Z}$,

$$\begin{aligned} f(x + m\mathbb{Z}) &\subseteq xf(1) + m\mathbb{Z} \\ &= ktf(1) + (ks)\mathbb{Z} \\ &\subseteq ktf(1) + k\mathbb{Z} \\ &\subseteq k\mathbb{Z}. \end{aligned}$$

□

Corollary 3.4. *For $n, m, k \in \mathbb{Z}^+$ and $(k,n)|(m,n)$. If $f \in \mathrm{Hom}(\mathbb{Z}_n, \circ_{m\mathbb{Z}_n})$, then $f(\overline{x} + m\mathbb{Z}_n) \subseteq k\mathbb{Z}_n$ for all $\overline{x} \in k\mathbb{Z}_n$.*

Theorem 3.5. *For $m, k \in \mathbb{Z}^+$ and $k|m$. Then*

$$k\mathbb{Z} = (1 \cdot k + m\mathbb{Z}) \cup \ldots \cup (ak + m\mathbb{Z}) = \bigcup_{i=1}^{a} (ik + m\mathbb{Z})$$

where $ak = m$.

Proof. Since $k|m$ and $m,k \in \mathbb{Z}^+$, $ak = m$ for some $a \in \mathbb{Z}^+$. For $x \in \mathbb{Z}$, by the Division Algorithm, $x = aq + r$ for some $q, r \in \mathbb{Z}$ and $0 \leq r < a$. Then

$$\begin{aligned} kx &= k(aq+r) \\ &= kaq + rk \\ &= mq + rk \in rk + m\mathbb{Z}. \end{aligned}$$

Therefore $k\mathbb{Z} \subseteq \bigcup_{i=1}^{a}(ik + m\mathbb{Z})$.
For $i = 1, \ldots a$, let $y \in ik + m\mathbb{Z}$. Then $y = ik + mt$ for some $t \in \mathbb{Z}$. Thus

$$y = ik + akt = k(i + at) \in k\mathbb{Z},$$

so $ik + m\mathbb{Z} \subseteq k\mathbb{Z}$. Therefore $\bigcup_{i=1}^{a}(ik + m\mathbb{Z}) \subseteq k\mathbb{Z}$.
Hence $k\mathbb{Z} = \bigcup_{i=1}^{a}(ik + m\mathbb{Z})$. □

Corollary 3.6. *For $n,m,k \in \mathbb{Z}^+$ and $(k,n)|(m,n)$. Then*

$$k\mathbb{Z}_n = (1 \cdot \overline{k} + m\mathbb{Z}) \cup \ldots \cup (a\overline{k} + m\mathbb{Z}_n) = \bigcup_{i=1}^{a}(i\overline{k} + m\mathbb{Z}_n)$$

where $a(k,n) = n$.

Theorem 3.7. *For $m,k \in \mathbb{Z}^+$, $k|m$ and $f \in \mathrm{Hom}(\mathbb{Z}, \circ_{m\mathbb{Z}})$. If g is an identity mapping, then $(f,g) \in \mathrm{SfHom}((F_k,\mathbb{Z}),(G_k,\mathbb{Z}))_{(\mathbb{Z},\circ_{m\mathbb{Z}})}$ if and only if $f(1)$ and m are relatively prime and $xf(1) + m\mathbb{Z} \subseteq f(x + m\mathbb{Z})$ for $x = 1,\ldots,m$.*

Proof. Let g be an identity map and $f \in \mathrm{Hom}(\mathbb{Z}, \circ_{m\mathbb{Z}})$.

Assume that $(f,g) \in \mathrm{SfHom}((F_k,\mathbb{Z}),(G_k,\mathbb{Z}))_{(\mathbb{Z},\circ_{m\mathbb{Z}})}$. So $f(\mathbb{Z}) = \mathbb{Z}$. Hence f is onto. By Theorem 2.2, $f(1)$ and m are relatively prime and $xf(1) + m\mathbb{Z} \subseteq f(x + m\mathbb{Z})$ for $x = 1,\ldots,m$.

Conversely, assume that $f(1)$ and m are relatively prime and $xf(1) + m\mathbb{Z} \subseteq f(x + m\mathbb{Z})$ for $x = 1,\ldots,m$, we will show that $f(F_k(x)) = G_k(g(x))$ for all $x \in \mathbb{Z}$.
If $F_k(x) = \mathbb{Z}$. By Theorem 2.2, $f(\mathbb{Z}) = \mathbb{Z}$.
If $F_k(x) = k\mathbb{Z}$. Then

$$\begin{aligned} f(k\mathbb{Z}) &= f\Big(\bigcup_{i=1}^{a}(ik + m\mathbb{Z})\Big) && \text{(by Theorem 3.5)} \\ &= \bigcup_{i=1}^{a} f(ik + m\mathbb{Z}) \\ &= \bigcup_{i=1}^{a}(ikf(1) + m\mathbb{Z}) && \text{by } f \in \mathrm{Hom}(\mathbb{Z}, \circ_{m\mathbb{Z}}) \text{ and } xf(1) + m\mathbb{Z} \subseteq f(x + m\mathbb{Z}). \end{aligned}$$

Since $f(1)$ and m are relatively prime, $if(1) + m\mathbb{Z} \neq jf(1) + m\mathbb{Z}$ for $i \neq j$. By Theorem 3.3, that is $f(x + m\mathbb{Z}) \subseteq k\mathbb{Z}$ for all $x \in k\mathbb{Z}$. We have $f(k\mathbb{Z}) = \bigcup_{i=1}^{a}(ikf(1) + m\mathbb{Z}) = k\mathbb{Z}$.
Therefore $(f,g) \in \mathrm{SfHom}((F_k,\mathbb{Z}),(G_k,\mathbb{Z}))_{(\mathbb{Z},\circ_{m\mathbb{Z}})}$. □

Corollary 3.8. *For an identity mapping g,*

$$f \in \operatorname{Epi}(\mathbb{Z}, \circ_{m\mathbb{Z}}) \text{ if and only if } (f,g) \in \operatorname{SfHom}((F_k,\mathbb{Z}),(G_k,\mathbb{Z}))_{(\mathbb{Z},\circ_{m\mathbb{Z}})}.$$

Theorem 3.9. *For $m,k,n \in \mathbb{Z}^+$, $(k,n)|(m,n)$ and $f \in \operatorname{Hom}(\mathbb{Z}_n, \circ_{m\mathbb{Z}_n})$. If g is an identity mapping, then $(f,g) \in \operatorname{SfHom}((F_k,\mathbb{Z}_n),(G_k,\mathbb{Z}_n))_{(\mathbb{Z}_n,\circ_{m\mathbb{Z}_n})}$ if and only if $f(\overline{1})$ and (m,n) are relatively prime and $xf(\overline{1}) + m\mathbb{Z}_n \subseteq f(\overline{x} + m\mathbb{Z}_n)$ for $x = 1,\ldots,\dfrac{n}{(m,n)}$.*

Proof. Let g be an identity map and $f \in \operatorname{Hom}(\mathbb{Z}_n, \circ_{m\mathbb{Z}_n})$.

Assume that $(f,g) \in \operatorname{SfHom}((F_k,\mathbb{Z}_n),(G_k,\mathbb{Z}_n))_{(\mathbb{Z}_n,\circ_{m\mathbb{Z}_n})}$. So $f(\mathbb{Z}_n) = \mathbb{Z}_n$. Hence f is an onto. By Theorem 2.4, $f(\overline{1})$ and (m,n) are relatively prime and $xf(\overline{1}) + m\mathbb{Z}_n \subseteq f(\overline{x} + m\mathbb{Z}_n)$ for $x = 1,\ldots,\dfrac{n}{(m,n)}$.

Conversely, assume that $f(\overline{1})$ and (m,n) are relatively prime and $xf(\overline{1}) + m\mathbb{Z}_n \subseteq f(\overline{x} + m\mathbb{Z}_n)$ for $x = 1,\ldots,\dfrac{n}{(m,n)}$, we will show that $f(F_k(x)) = G_k(g(x))$ for all $x \in \mathbb{Z}$.
If $F_k(x) = \mathbb{Z}_n$. By Theorem 2.4, $f(\mathbb{Z}_n) = \mathbb{Z}_n$.
If $F_k(x) = k\mathbb{Z}_n$. Then

$$\begin{aligned} f(k\mathbb{Z}_n) &= f\Big(\bigcup_{i=1}^{a}(i\overline{k} + m\mathbb{Z}_n)\Big) && \text{(by Corollary 3.6)} \\ &= \bigcup_{i=1}^{a} f(i\overline{k} + m\mathbb{Z}_n) \\ &= \bigcup_{i=1}^{a}(i\overline{k}f(\overline{1}) + m\mathbb{Z}_n) && \text{by } f \in \operatorname{Hom}(\mathbb{Z}, \circ_{m\mathbb{Z}_n}) \text{ and } xf(\overline{1}) + m\mathbb{Z}_n \subseteq f(\overline{x} + m\mathbb{Z}_n). \end{aligned}$$

Since $f(\overline{1})$ and (m,n) are relatively prime, $if(\overline{1}) + m\mathbb{Z}_n \neq jf(\overline{1}) + m\mathbb{Z}_n$ for $i \neq j$. By Corollary 3.4, that is $f(\overline{x} + m\mathbb{Z}_n) \subseteq k\mathbb{Z}_n$ for all $\overline{x} \in k\mathbb{Z}_n$. We have $f(k\mathbb{Z}_n) = \bigcup_{i=1}^{a}(ikf(\overline{1}) + m\mathbb{Z}_n) = k\mathbb{Z}_n$.
Therefore $(f,g) \in \operatorname{SfHom}((F_k,\mathbb{Z}_n),(G_k,\mathbb{Z}_n))_{(\mathbb{Z}_n,\circ_{m\mathbb{Z}_n})}$. □

Corollary 3.10. *For an identity mapping g,*

$$f \in \operatorname{Epi}(\mathbb{Z}_n, \circ_{m\mathbb{Z}_n}) \text{ if and only if } (f,g) \in \operatorname{SfHom}((F_k,\mathbb{Z}_n),(G_k,\mathbb{Z}_n))_{(\mathbb{Z}_n,\circ_{m\mathbb{Z}_n})}.$$

Example 3.11. *Consider $m = 12$, $f \in \operatorname{Hom}(\mathbb{Z}, \circ_{12\mathbb{Z}})$ so (f,g) is a soft inclusion homomorphism from $(F_4,\mathbb{Z})$ into $(G_4,\mathbb{Z})$ where $f(1) = 5$, $5x + 12\mathbb{Z} \subseteq f(x + 12\mathbb{Z})$ and g is an identity map.*

If $5x + 12\mathbb{Z} \nsubseteq f(x + 12\mathbb{Z})$ then $f(x + 12\mathbb{Z}) \neq 5x + 12\mathbb{Z}$. So

$$\begin{aligned} f(4\mathbb{Z}) &= f\Big(\bigcup_{i=1}^{3}(4i + 12\mathbb{Z})\Big) \\ &= \bigcup_{i=1}^{3} f(4i + 12\mathbb{Z}) \\ &\neq \bigcup_{i=1}^{3}(4if(1) + 12\mathbb{Z}) \\ &= (8 + 12\mathbb{Z}) \cup (4 + 12\mathbb{Z}) \cup (12\mathbb{Z}) = 4\mathbb{Z}. \end{aligned}$$

Hence $f(4\mathbb{Z}) \neq 4\mathbb{Z}$ and $(f,g) \notin \text{SfHom}((F_4,\mathbb{Z}),(G_4,\mathbb{Z}))_{(\mathbb{Z},\circ_{12\mathbb{Z}})}$.

If $f(1) = 3$ then $f(\mathbb{Z}) \neq \mathbb{Z}$ since $\mathbb{Z} = \bigcup_{i=1}^{11}(i+12\mathbb{Z})$

$$\begin{aligned} f(\mathbb{Z}) &= f(\bigcup_{i=1}^{11}(i+12\mathbb{Z})) \\ &= \bigcup_{i=1}^{11} f(i+12\mathbb{Z}) \\ &= \bigcup_{i=1}^{11}(if(1)+12\mathbb{Z}) \\ &= (3+12\mathbb{Z}) \cup (6+12\mathbb{Z}) \cup (9+12\mathbb{Z}) \cup (12\mathbb{Z}) \neq \mathbb{Z}. \end{aligned}$$

Hence $(f,g) \notin \text{SfHom}((F_4,\mathbb{Z}),(G_4,\mathbb{Z}))_{(\mathbb{Z},\circ_{12\mathbb{Z}})}$.

In conclusion, we will see that soft homomorphisms on $(\mathbb{Z},\circ_{m\mathbb{Z}})$ preserves k-soft hypergroups that is the following corollary.

Corollary 3.12. *For $m \in \mathbb{Z}^+$ and g is an identity mapping. If $(f,g) \in \text{SfHom}((F_k,\mathbb{Z}),(G_l,\mathbb{Z}))_{(\mathbb{Z},\circ_{m\mathbb{Z}})}$, then $k = l$.*

Acknowledgement(s) : I would like to thank the referee(s) for comments and suggestions on the manuscript. This work was supported by the National Research Council of Thailand and Mathematical Association of Thailand.

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