Research Article

Designing of Double Acceptance Sampling Plan for Zero-inflated and Over-dispersed Data Using Multi-objective Optimization

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Abstract
The double acceptance sampling plan (*DSP*) is wildly used tools for the decision of production quality control. In actually, most production processes have excellent quality control and well inspected, the number of defective items for many samples will be zero. For this reason, the traditional probability distribution is not appropriate for the *DSP*. This research proposed the *DSP* for the manufacturing that was affected by zero-inflated and over-dispersed count data. The number of defects for a sample inspection is considered under the zero-inflated Negative Binomial (*ZINB*) distribution. The required sample sizes (n_1, n_2) are designed to achieve the optimal plan parameter of $(n_1, n_2, c_1, c_2)^*$ the *DSP* under the *ZINB* distribution (DSP_{ZINB}). The Genetic Algorithm with multi-objective optimization is used to estimate the optimal plan parameters which are maximizing the probability of accepting a lot (P_a) and minimizing the total cost of inspection (*TC*) and the average number of samples (*ASN*) simultaneously. The sensitivity analysis of the required sample size is used to analyze the performance of the proposed DSP_{ZINB} which is presented through three numerical examples. The results showed that a smaller of required sample sizes and $n_1 < n_2$ are provide the optimal plan parameters to achieve the minimum and maximum value of the multi-objective function. Moreover, the proposed DSP_{ZINB} give a good performance when a shape parameter of *ZINB* distribution (k) is small and approaches zeros while a zero-inflation parameter (ϕ) is a large value.

Keywords: The double acceptance sampling plan, Zero-inflated negative binomial distribution, Over-dispersion, Genetic algorithm, Multi-objective optimization

1 Introduction

The one important tool in the product control technique is an acceptance sampling plan (*ASP*). The *ASP* is applied in many departments to inspect the quality such as raw materials, some partial products of the production process, and finished products. This technique helps consumers decide to accept or reject a product which is produced by manufacturers based on sampling results selected from a lot. Users can decide to choose the minimum sample size from a sampling plan to achieve the acceptance criteria and not acceptable for that lot. Dodge and Romig's tables are widely used to decide for a sampling plan in which users know the lot size,

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percent nonconforming of a lot, producer's risk, and consumer's risk for the production process [1]. The single sampling plan (*SSP*) is easy to use but usually results in a larger average number of items inspected than the other sampling plan such as the *DSP* [2]. Moreover, manufacturers prefer to use the *DSP* for sampling inspection than the *SSP*, if they want to make a clear and precise decision on acceptance or rejection lots. In practice, some necessary values are unknown, then they cannot choose the optimal *ASP*. This problem can affect the total cost of the inspection.

Recently, many researchers have studied the determination of optimal *ASP* model using optimization techniques as follows. Duarte and Saraiva [3] proposed a method to find the optimal value of the *ASP* model. The objective function is finding the lowest value of error for the probability of accepting the *ASP* model for single and double sampling plans corresponding to the sample size and the acceptable number. Kaya [4] uses a Genetic Algorithm (GA) to determine the sample size of the attribute control chart for a multi-state process in which the objective function is finding the minimum cost and the maximum probability of accepting the model. Kobilinsky and Bertheau [5] presented a cost function for the inspection process depend on the number of inspection groups and sample sizes for single and double which are based on the manufacturer's risk and the consumer's risk. Cheng and Chen [6] applied the GA methods to design the *DSP*. These models increase the efficiency of the design *ASP* and reduce errors of the manufacturer's risk and consumer's risk. Moreover, GA methods can help to find the best information more efficiently and more accurately. Sampath and Deepa [7] presented the *DSP* using the GA method to determine the optimal sample size and acceptance number under the manufacturer's risk and the consumer's risk. Braimah *et al*.[8] evaluated the optimal value of the mathematical model to determine the sample size and the random range for the *ASP*. The condition of the model is the acceptance number equal to zero.

There are 3 important objectives that the manufacturer expects to achieve from the optimal *ASP*: the lowest cost, the smallest *ASN*, and the highest probability of acceptance. Also, some researchers have studied the economic model of various *ASPs*, such as, Hsu [9] proposed the cost model of the single *ASP* to evaluate the minimum cost that is appropriate for both manufacturers and consumers. The study found that the proposed cost model is used to inspect a defective of items. Fallahnezhad and Aslam [10] designed an economic model of the *ASPs* under Bayesian inference. The decision to receive the lot depends on the proposed model. Fallahnezhad and Qazvini[11] designed a new economic model of the *ASP* in a two-stage approach based on the Maxima Nomination Sampling (MNS) technique. Fallahnezhad *et al*. [12] presented the *ASP* based on the MNS method in the current inspection errors. An economical model is proposed in terms of inspection errors and investigated the impact of errors from an economical point of view.

Currently, most production processes have excellent quality control. It was found that when the production process is well inspected, the zero defects are more discover in sample inspections. For this reason, some researchers presented that a zero-inflated Poisson (*ZIP*) distribution is appropriated for the probability distribution of the number of defects. There are some researchers, such as [13]–[16], designed the inspection process when the number of defective items is a *ZIP* distribution. Also, the optimal plan parameters under several sampling plans are presented.

In the real situation, count data are zero-inflation (extra zeroes) and overdispersion (variance larger than mean). So, some researchers such as Ridout *et al*. [17] and Fang [18] study the performance between the model of *ZIP* regression and *ZINB*regression where the count data are overdispersed. They claim that a *ZINB* distribution is more flexible for *ZIP* distribution. Arianna *et al*. [19] study the microbial data characterized by an excess of zero counts based on *ZIP* distribution and *ZINB* distribution. Wang and Hailemariam [20] present three new sampling plans when the number of samples in the food industry excess of zeros. The performance of the proposed sampling plan is studied under the *ZINB* distribution.

The above-mentioned research can be interpreted that three important objectives function that the manufacturers expect to achieve from the optimal *ASP*: the highest probability of acceptance, the lowest cost, and the smallest *ASN*. This research aims to design the required sample sizes to achieve the optimal plan parameter of the proposed DSP_{ZINB} under three objective functions simultaneously. The sensitivity analysis is used to evaluate the performance of the proposed DSP_{ZINB}. A method of the GA with multi-objective optimization is applied for the simulation study using MATLAB sofware [21]. The rest of paper is organized as follows: Section 2 explains the brief concept of the *ZINB* distribution, the method of designing the proposed DSP_{ZINB}, and the total cost function. Simulation results are presented and analyzed in Section 3. Finally, conclusions are presented in Section 4.

2 Material and Methods

2.1 *Zero-inflated Negative Binomial distribution*

Currently, the number of defective items for many samples will be zero when most production processes have excellent quality control and the production process is well inspected. Under the above situation, the proper probability distribution function of the number of defective items for sample inspection is the Zero-Inflated (*ZI*) distribution. The *ZI* distribution is a mixture between the process generates zeros and the other processes that are a count distribution under non-negative integers. Suppose *X* is a random variable under the *ZI* distribution, then the probability mass function (pmf) of Xis given by [Equations (1) and (2)]

$$P(X = x|\phi,\Theta) = \phi f(x) + (1-\phi)g(x;\Theta) \quad (1)$$

$$\text{where } f(x) = \begin{cases} 1, & x = 0 \\ 0, & x = 1,2,3,... \end{cases} \quad (2)$$

Let ϕ be a zero-inflation parameter, $0 < \phi < 1$, and $g(x; \Theta)$is the probability mass function (pmf) of *X* with a vector of the parameter, $\Theta = \{\theta_1, \theta_2, \ldots, \theta_n\}$.

In this study, we consider zero-inflated count models corresponding to the Negative-Binomial (*NB*) distribution called *ZINB* distribution. The *NB* distribution is given by Arianna *et al*. [19], the pmf of *NB* distribution is given as Equation (3).

$$g(x;\mu_{NB},k) = \frac{\Gamma(x+k)}{\Gamma(x+1)\Gamma(k)}\left(\frac{k}{k+\mu_{NB}}\right)^k \times \left(\frac{\mu_{NB}}{k+\mu_{NB}}\right)^x \; ; \; x = 0,1,2,... \quad (3)$$

where *x* is failure that occurs in a sample unit and $\Gamma(.)$ is the complete gamma function. Let *k* be the shape parameter that quantifies the amount of over-dispersion, the mean and variance for *NB* distribution are $\mu_{NB} = k_p$ and $\sigma^2_{NB} = \mu_{NB} + \frac{\mu^2_{NB}}{k}$, respectively.

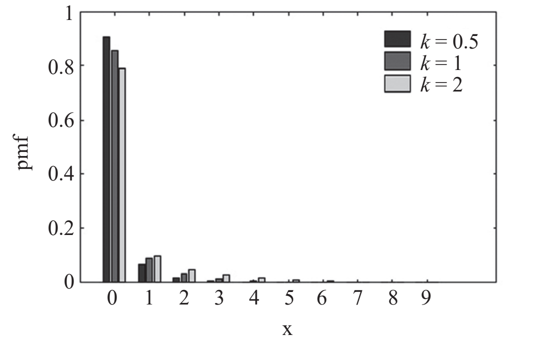

Figure 1: The pmf of *ZINB* distribution under $\phi = 0.001$, $k = 0.50$,1, and 2.

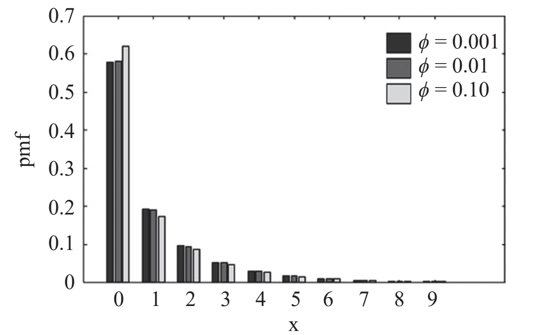

Figure 2: The pmf of *ZINB* distribution under $k = 0.5$, $\phi = 0.001$, 0.01, and 0.10.

The *ZINB* distribution is given by Wang and Hailemariam [20], which is a special type of mixer between Bernoulli distribution and Negative Binomial distribution. From the pmf of *NB* distribution, then the pmf of *ZINB* distribution has the form Equation (4).

$$P(X = x|\mu_{NB},k,\phi,) = \begin{cases} \phi + (1-\phi)\left(\frac{k}{k+\mu_{NB}}\right)^k, & x = 0 \\ (1-\phi)\left(\frac{k}{k+\mu_{NB}}\right)^k \times \left(\frac{\mu_{NB}}{k+\mu_{NB}}\right)^x, & x = 1,2,.. \end{cases} \quad (4)$$

where $0 < \phi < 1$. Moreover, the mean and variance of *ZINB* the distribution are $\mu_{ZINB} = (1 - \phi)\,\mu_{NB}$ and $\sigma^2_{ZINB} = \mu_{ZINB}(1 + p + \lambda\phi)$ respectively. Figures 1 and 2 presented the pmf of *ZINB* distribution under the difference value of k = (0.50, 1, 2) and ϕ = (0.001, 0.01, 0.10).

2.2 *Designed of the double acceptance sampling plan for zero-inflated Negative Binomial distribution*

The double sampling plan (*DSP*) requires the specification of four quantities which are known as its parameters. These parameters are n_1, n_2, c_1 and c_2 respectively. In a double sampling plan, the decision of acceptance or rejection of the lot is taken based on two samples.

1) The lot is accepted in the first sample if the number of defective units (d_1) in the first sample is less than the acceptance number c_1.

2) The lot is accepted in the second sample if the number of defective units ($d_1 + d_2$) in both samples is greater than c_1 and less than or equal to the acceptance number c_2.

Let P_a^1 and P_a^2 denote the probabilities of accepting a lot on the first sample and the second sample as shown in Equations (5) and (6) respectively, then the probability of accepting a lot P_a of proportion of defective per lot p is given by Equation (7).

$$P_a^1(p) = P(d_1 \le c_1 : n_1) \tag{5}$$

$$P_a^2(p) = P(c_1 < d_1 \le c_2 : n_1) \times P(d_1 + d_2 \le c_2 : n_2) \tag{6}$$

$$P_a(p) = P_a^1(p) + P_a^2(p) \tag{7}$$

In this section, the optimal *DSP* under the *ZINB* distribution (DSP_{ZINB}) is described. By applied Equation (4) and Equation (7), the probability of accepting a lot for *ZINB* distribution is shown in Equation (8).

$$\begin{aligned}
P_{a,ZINB}(p) &= \phi + (1-\phi)\left(\frac{k}{k+\mu_{NB_1}}\right)^k \\
&+ \sum_{d_1=1}^{c_1}(1-\phi)\frac{\Gamma(d_1+k)}{\Gamma(d_1+1)\Gamma(k)}\left(\frac{k}{k+\mu_{NB_1}}\right)^k\left(\frac{\mu_{NB_1}}{k+\mu_{NB_1}}\right)^{d_1} \\
&+ \sum_{d_1=c_1+1}^{c_2}\left\{\left[(1-\phi)\frac{\Gamma(d_1+k)}{\Gamma(d_1+1)\Gamma(k)}\left(\frac{k}{k+\mu_{NB_1}}\right)^k\left(\frac{\mu_{NB_1}}{k+\mu_{NB_1}}\right)^{d_1}\right]\right. \\
&\times\left[\phi + (1-\phi)\left(\frac{k}{k+\mu_{NB_2}}\right)^k\right. \\
&\left.\left.+ \sum_{d_2=1}^{c_2-d_1}(1-\phi)\frac{\Gamma(d_2+k)}{\Gamma(d_2+1)\Gamma(k)}\left(\frac{k}{k+\mu_{NB_2}}\right)^k\left(\frac{\mu_{NB_2}}{k+\mu_{NB_2}}\right)^{d_2}\right]\right\}
\end{aligned} \tag{8}$$

where $\mu_{NB1} = k_{p1}$ and $\mu_{NB2} = k_{p2}$. Let P_I be the probability of deciding on the acceptance or rejection of the lot on the first sample and is given by [Equation (9)].

$$P_I = P(d_1 \le c_1 : n_1) + P(d_1 > c_2 : n_1). \tag{9}$$

Appling Equation (8) for *ZINB* distribution, the *ASN* function of the DSP_{ZINB} is given by [Equation (10)].

$$\begin{aligned}
ASN &= n_1 + n_2(1-P_I) \\
&= n_1 + n_2\left[\sum_{d_1=c_1+1}^{c_2}(1-\phi)\frac{\Gamma(d_1+k)}{\Gamma(d_1+1)\Gamma(k)}\left(\frac{k}{k+\mu_{NB_1}}\right)^k\right. \\
&\left.\times\left(\frac{\mu_{NB_1}}{k+\mu_{NB_1}}\right)^{d_1}\right]
\end{aligned} \tag{10}$$

2.3 *The total cost function of double acceptance sampling plan*

In this section, the total cost function in product inspection a lot for the DSP_{ZINB} is proposed. Three different types of costs are considered. The component of the total cost function for inspection a lot for the DSP_{ZINB} can be expressed as follow:

First-component: C_I denotes the cost of inspection per lot as following in Equation (11).

$$C_I = C_I \cdot \left(n_1 P_a^1(p) + (n_1+n_2)P_a^2(p) + N(1-P_a(p))\right) \tag{11}$$

where C_I represents the inspection cost per unit and $n_1 P_a^1(p) + (n_1+n_2)P_a^2(p) + N(1-P_a(p))$ represents the expected number of units inspected per lot respectively. Three terms of the expected number of units inspected per lot can be explained as follows:

- $n_1 P_a^1(p)$: This term denotes the expected number of units inspected if the lot is accepted in the first inspection of the .
- $(n_1+n_2)P_a^2(p)$: This term denotes the expected number of units inspected if the lot is accepted in the second inspection of the *DSP*.
- $N(1-P_a(p))$ This term denotes the expected number of units inspected if the lot is rejected with the probability $1 - P_a(p)$ of the *DSP*.

Second-component: C_F denotes the cost of the internal failure per lot as following in Equation (12).

$$C_F = C_2\left(\left(n_1+n_2\right)p+\left(1-P_a\left(p\right)\right)\left(N-\left(n_1+n_2\right)\right)p\right)$$
$$= C_2 p\left(\left(n_1+n_2\right)+N\left(1-P_a\left(p\right)\right)-\left(n_1+n_2\right)\left(1-P_a\left(p\right)\right)\right)$$
$$= C_2 p\left(N-NP_a\left(p\right)+\left(n_1+n_2\right)P_a\left(p\right)\right)$$
$$= C_2\cdot Np\left(1-P_a\left(p\right)+P_a\left(p\right)\frac{\left(n_1+n_2\right)}{N}\right) \quad (12)$$

where C_2 represents the internal failure cost per unit and $Np\left(1-P_a\left(p\right)+P_a\left(p\right)\frac{\left(n_1+n_2\right)}{N}\right)$ represents the expected number of defective items detected per lot respectively. Two terms of the expected number of defective items detected per lot can be explained as follows:

- $(n_1 + n_2)p$: This term denotes the expected number of defective items detected if the inspection of the is 100%, for the sampled $n_1 + n_2$ items.
- $(1- P_a(p)\ (N-(n_1+n_2))$: This term denotes the expected number of defective items detected if the lot is rejected with probability $1- P_a(p)$, it will be 100% inspected and the remaining $(N-(n_1+n_2))p$ defective items will be detected.

Third-component: C_o denotes the cost of an outgoing defective per lot as following in Equation (13).

$$C_o = C_3\cdot P_a\left(p\right)\left(N-\left(n_1+n_2\right)\right)p \quad (13)$$

where C_3 represents the cost of an outgoing defective per unit and $P_a(p)\ (N-(n_1+n_2))$ represents the expected number of defective items not detected per lot. This term can be explained that if the lot is accepted with probability $P_a(p)$, the defective items will not be detected is $(N-(n_1+n_2))p$.

Therefore, the total cost of inspection a lot for the DSP_{ZINB} can be expressed in Equation (14).

$$TC = C_I + C_F + C_o$$
$$= C_1\cdot\left(n_1P_a^1\left(p\right)+\left(n_1+n_2\right)P_a^2\left(p\right)+N\left(1-P_a\left(p\right)\right)\right)$$
$$+C_2\cdot Np\left(1-P_a\left(p\right)+P_a\left(p\right)\frac{\left(n_1+n_2\right)}{N}\right) \quad (14)$$
$$+C_3\cdot P_a\left(p\right)\left(N-\left(n_1+n_2\right)\right)p$$

3 Results and Discussion

In this section, the optimal plan parameters $(n_1,n_2,c_1,c_2)^*$ of the proposed DSP_{ZINB} are calculated to achieve the minimum and maximum value of multi-objective function simultaneously. MATLAB software is used in a simulation study in a method of the GA with multi-objective optimization. The constraints of the producer's risk (α) and the consumer's risk (β) are satisfied immediately for the provided of the acceptable quality level (AQL) and the lot tolerance percent defective ($LTPD$). For the effectiveness of the proposed sampling plan, two points (AQL, $1-\alpha$) and ($LTPD$, β) are considered for changes on the OC curve. A manufacturer intends that the occasion of the probability of accepting a lot should be greater than $1-\alpha$ at the quality level of AQL. In real cause, a customer requests that the probability of accepting a lot should be less than β at $LTPD$.

In the optimization technique, the optimal solution is considered on three objective functions concurrently, as follows.

Multi-objective function:

Minimize TC and ASN (15)

Maximize $P_a(p)$ (16)

Subject to: $n_1 + n_2 \le \delta N$,

$$n_1 > 0, n_2 > 0, c_1 \ge 0, c_2 > 0$$
$$P_a\left(AQL\right)\ge 1-\alpha \text{ and } P_a\left(LTPD\right)\le\beta$$

In fact that the required sample sizes (n_1,n_2) under the DSP for inspection in the production process have the most effect on cost. Assume that the following sets of input parameters are given:

$N = 1{,}000$, $\alpha = 0.05$, and $\beta = 0.01$ respectively. The fixed value of AQL is 0.01 and 0.05, is 0.05,0.075,0.1. Also, the fixed value of cost in each status are $C_I + 1$, $C_F = 2$, and $C_o = 10$ respectively.

3.1 *Numerical example 1*

Equations (10) and (14) show the value of TC and ASN of the proposed DSP_{ZINB} under the required sample sizes (n_1,n_2). Therefore, the sensitivity analysis for the optimal value of the required sample sizes (n_1,n_2) is an important situation in the manufacturing process. In this numerical example, the required sample sizes (n_1,n_2) are considered by assuming that δ is the proportion of the sample sizes from the lot size, $\delta = \frac{n_1+n_2}{N}$.

Furthermore, the comparison between the size of the first sample (n_1) and the second sample (n_2) is considered. Also, there are three different scenarios of the required sample sizes (n_1,n_2) to find the optimal value under multi-objective function of the DSP_{ZINB} as follows.

Scenario 1 (S1): $n_1 = n_2$
Scenario 2 (S2): $n_1 < n_2$
Scenario 3 (S3): $n_1 > n_2$

Three scenarios of the required sample size are used to measure discrimination of the proposed DSP_{ZINB}. Depends on the above scenarios, the following multi-objective optimization problem is solving to investigate the optimal parameters $(n_1,n_2,c_1,c_2)^*$ for the proposed DSP_{ZINB} using GA optimization. Suppose that the proportion of defective for a lot is $p = 0.05$ under the different combinations of $k = 0.50$, $AQL = 0.05$, $LTPD = 0.10$, $\delta = (0.05, 0.10, 0.20, 0.25)$ and (0.001,0.01,0.05,0.09,0.10).

From Table 1, the sensitivity analyses of optimal parameters $(n_1,n_2,c_1,c_2)^*$ under the proposed DSP_{ZINB} are shown by considering three conditions of the required sample sizes. The maximum value of $P_a(p)$ and the minimum value of TC and ASN can be determined by solving Equations (15) and (16), with a given value of p,k,ϕ, AQL and $LTPD$, using GA multi-objective optimization. The optimal plan parameters $(n_1,n_2,c_1,c_2)^*$ under the proposed DSP_{ZINB} are determined by satisfying 2 inequalities, $P_a(AQL) \geq 1 - \alpha$ and $P_a(LTPD) \leq \beta$. The investigating values are given as follows.

1) Based on the considering that the sample sizes for the proposed DSP_{ZINB} are determined to be less than or equal to 5% of lot size ($\delta = 0.05$) under the same value of k, ϕ, AQL and $LTPD$. The result shows that,

Table 1: The effect of ϕ on the performances of DSP_{ZINB} under three conditions of the required sample size

δ	ϕ	$n_1 = n_2$			$n_1 < n_2$			$n_1 > n_2$		
		(25,25,0,1)*			**(17,33,0,1)***			**(33,17,0,1)***		
		P_a	TC	ASN	P_a	TC	ASN	P_a	TC	ASN
0.05	0.001	0.9759	516	26.82	0.9759	509	19.41	0.9759	524	34.24
	0.01	0.9761	516	26.81	0.9761	508	19.38	0.9761	524	34.23
	0.05	0.9771	515	26.73	0.9771	508	19.29	0.9771	523	34.18
	0.09	0.9781	514	26.66	0.9781	507	19.19	0.9781	522	34.13
	0.10	0.9783	514	26.64	0.9783	506	19.17	0.9783	522	34.12
δ	ϕ	**(50,50,0,1)***			**(33,67,0,1)***			**(67,33,0,1)***		
		P_a	TC	ASN	P_a	TC	ASN	P_a	TC	ASN
0.10	0.001	0.9759	525	53.65	0.9759	508	37.89	0.9759	541	69.41
	0.01	0.9761	525	53.61	0.9761	508	37.84	0.9761	541	69.38
	0.05	0.9771	524	53.47	0.9771	507	37.65	0.9771	540	69.29
	0.09	0.9781	523	53.32	0.9781	506	37.45	0.9781	539	69.19
	0.10	0.9783	523	53.28	0.9783	502	37.40	0.9783	539	69.17
δ	ϕ	$n_1 = n_2$			$n_1 < n_2$			$n_1 > n_2$		
		(100,100,0,1)*			**(67,133,0,1)***			**(133,67,0,1)***		
		P_a	TC	ASN	P_a	TC	ASN	P_a	TC	ASN
0.20	0.001	0.9759	534	107.29	0.9759	501	76.70	0.9759	566	137.89
	0.01	0.9761	533	107.23	0.9761	501	76.61	0.9761	566	137.84
	0.05	0.9771	533	106.93	0.9771	500	76.22	0.9771	565	137.65
	0.09	0.9781	532	106.64	0.9781	499	75.83	0.9781	564	137.45
	0.10	0.9783	532	106.57	0.9783	499	75.74	0.9783	564	137.40
δ	ϕ	**(125,125,0,1)***			**(75,175,0,1)***			**(175,75,0,1)***		
		P_a	TC	ASN	P_a	TC	ASN	P_a	TC	ASN
0.25	0.001	0.9759	542	134.12	0.9759	493	87.76	0.9759	591	180.47
	0.01	0.9761	542	134.03	0.9761	493	87.65	0.9760	591	180.42
	0.05	0.9771	541	133.67	0.9771	492	87.14	0.9771	590	180.20
	0.09	0.9781	540	133.30	0.9781	491	86.62	0.9781	589	179.98
	0.10	0.9783	540	133.21	0.9783	491	86.50	0.9783	589	179.93

at ϕ = 0.001, the maximum value of the probability of accepting a lot for all scenarios is the same value $P_a(0.05) = 0.9759$. In addition, under the same condition, the S2 gives the minimum value of *TC* and *ASN* with optimal plan parameters $(17,33,0,1)^*$.

2) When ϕ increases, under the same value of ϕ, *p*, *AQL* and *LTPD*, the results show that the value of $P_a(0.05)$ tends to increase but the value of *TC* and *ASN* tends to decrease respectively. Furthermore, the S2 still provides the most optimal plan parameters.

3) The results indicate that $\delta = 0.05$ is given a lower value of *TC* and *ASN* than $\delta = 0.10, 0.20$ and 0.25 with the same value of $P_a(0.05)$ for all three scenarios. Other than that, the S2 provides the most optimal plan parameters to achieve the maximum value of $P_a(0.05)$, and the minimum value of *TC* and *ASN*.

It can interpret that the smaller of required sample sizes (lower δ) provides the optimal plan parameters of the proposed DSP_{ZINB} to achieve the maximum value of P_a, and the minimum value of *TC* and *ASN*.

3.2 *Numerical example 2*

In the general sampling system, the manufacturer expects that the smaller value of the required sample sizes (n_1,n_2) or *ASN* would be more satisfactory for designing the optimal *ASP*. For this reason, the numerical example aims to find the optimal plan parameters of the proposed DSP_{ZINB} along with satisfying under the fixed two-level values of ϕ, *AQL*, *LTPD*, and *p* as shown in Table 2.

In this numerical example, the sensitivity analysis of the optimal plan parameter is considered based on $n_1+n_2 \leq 100$ and k=0.50.

From Table 3, the result shows that the level of value (ϕ, *p*, *AQL*, *LTPD*) = (L, L, H, H) provides the optimal plan parameter $(43,57,0,9)^*$ that gives the minimum value of *TC* as 140 and *ASN* as 43.01. Other than that, the level of value (ϕ, *p*, *AQL*, *LTPD*) = (H,L, H, L) provides the optimal plan parameter $(44,56,0,11)^*$ that gives the minimum value of *TC* as 140 and *ASN* as 44.01 respectively.

For the result in Table 3, it can interpret that the S2 ($n_1<n_2$) proposed the optimal plan parameter which achieves the optimal solution of $P_a(p)$, *TC* and *ASN* while the value of c_2 is a very different from c_1. Moreover, It is seen that the proposed DSP_{ZINB} gives the optimal plan parameter when *LTPD* = 0.10.

Table 2: the two-level values of ϕ, *AQL*, *LTPD* and *p* under the proposed DSP_{ZINB}

Fixed Parameter	Low Values (L)	High Values (H)
ϕ	0.001	0.10
p	0.01	0.05
AQL	0.01	0.05
LTPD	0.05	0.10

3.3 *Numerical example 3*

In the real case, it was found that most of the inspection processes determine that the first sample and the second sample are equal ($n_1 = n_2$). So, in this example, the performance of the proposed DSP_{ZINB} with a different value of *k* under the optimal plan parameter $(50,50,0,1)^*$ is considered to achieve the optimal solution of $P_a(p)$, *TC*, and *ASN* as presented in Table 4. Figures 3–5 illustrate the OC curves, the *TC* curves, and the *ASN* curves under the proposed DSP_{ZINB} when considering the different values of *k* (*k* = 0.50,0,1,2). Figures 6–8 show that the OC curves, the *TC* curves, and the *ASN* curves under the proposed DSP_{ZINB} with a different value of ϕ (ϕ=0.01,0.05,010) under the optimal plan parameter $(50,50,0,1)^*$.

Table 3: The optimal parameters $(n_1,n_2,c_1,c_2)^*$ for DSP_{ZINB} based on $n_1+n_2 \leq 100$ and $k = 0.50$

ϕ	*p*	*LTPD*	*AQL*	Optimal Parameters				Optimal Solution		
				n_1	n_2	c_1	c_2	P_a	*TC*	*ASN*
L	L	L	L	49	51	0	1	0.9950	146	49.79
L	L	H	H	***43***	***57***	***0***	***9***	***0.9953***	***140***	***43.01***
L	H	L	H	49	51	0	1	0.9759	524	52.72
L	H	H	L	43	57	0	11	0.9816	513	43.01
H	L	L	H	48	52	0	1	0.9955	145	49.71
H	L	H	L	***44***	***56***	***0***	***11***	***0.9957***	***140***	***44.01***
H	H	L	L	49	51	0	1	0.9783	522	52.35
H	H	H	H	28	72	0	14	0.9830	497	28.02

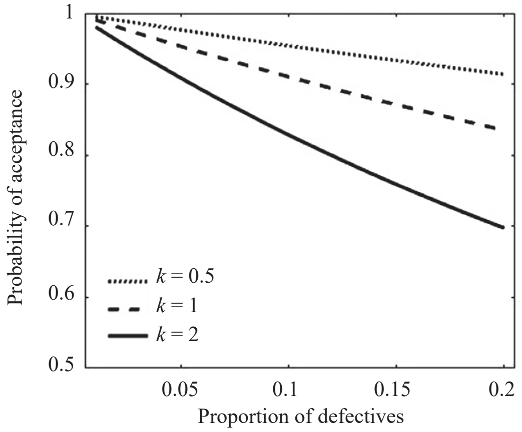

Figure 3: The optimal OC function under the optimal plan parameter (50, 50, 0, 1)* and k = 0.50, 1.2.

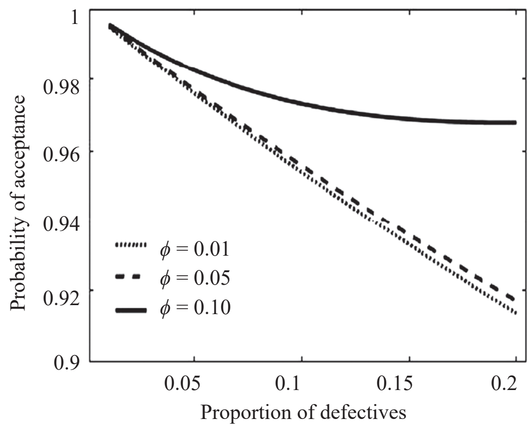

Figure 6: The optimal OC function under the optimal plan (50, 50, 0, 1)* and ϕ = 0.01, 0.05, 0.10.

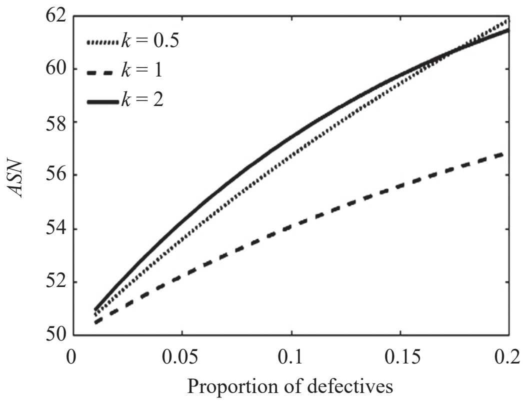

Figure 4: The optimal *ASN* curves under the optimal plan parameter (50, 50, 0, 1)* and k = 0.50, 1.2.

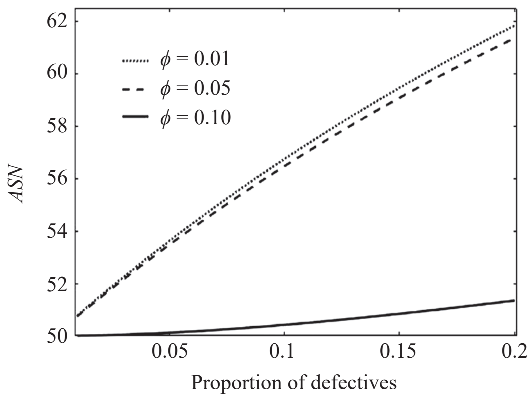

Figure 7: The optimal *ASN* curves under the optimal plan (50, 50, 0, 1)* and ϕ = 0.01, 0.05, 0.10.

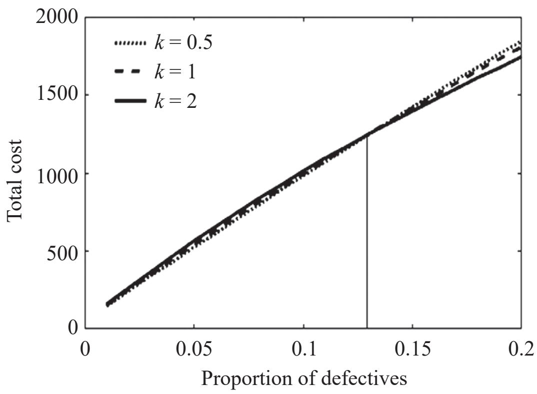

Figure 5: The optimal *TC* curves under the optimal plan parameter (50, 50, 0, 1)* and k = 0.50,1.2.

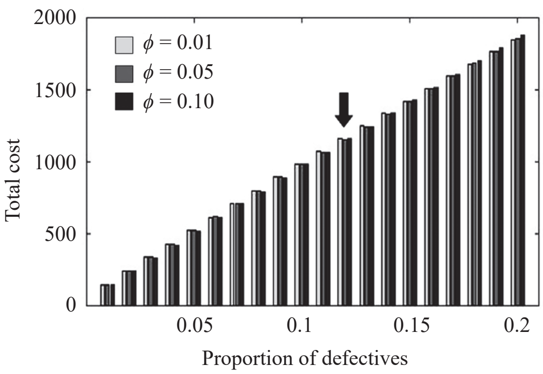

Figure 8: The optimal *TC* curves under the optimal plan (50, 50, 0, 1)* and ϕ = 0.01, 0.05, 0.10.

Table 4: The effect of *k* on the performances of DSP_{ZINB} under the optimal required plan parameter $(50,50,0,1)^*$, *AQL* = 0.05, and *LTPD* = 0.10

p	ϕ	*k* = 0.50			*k* = 1			*k* = 2		
		P_a	*TC*	*ASN*	P_a	*TC*	*ASN*	P_a	*TC*	*ASN*
0.01	0.001	0.9950	147	50.77	0.9901	151	50.49	0.9803	161	50.97
	0.05	0.9953	146	50.74	0.9906	151	50.47	0.9813	160	50.92
	0.10	0.9955	146	50.70	0.9911	150	50.44	0.9823	159	50.87
	0.20	0.9960	146	50.62	0.9921	150	50.39	0.9842	157	50.78
0.05	0.001	0.9759	525	53.65	0.9524	539	52.27	0.9071	566	54.31
	0.05	0.9771	524	53.47	0.9548	537	52.15	0.9117	562	54.10
	0.10	0.9783	523	53.28	0.9571	535	52.04	0.9163	557	53.89
	0.20	0.9807	520	52.92	0.9619	530	51.81	0.9256	557	53.46

4 Conclusions

Nowadays, It was found that when the production process is well inspected, the zero defects are more detect in sample inspections. There are many ways to achieve the optimal *DSP* that is affected by zero-inflated and overdispersed data. In this research, the proposed method is modified to make an optimal decision for the manufacturer. The optimal plan parameters are proposed to the DSP_{ZINB}, which are calculated to achieve the minimum and maximum value of multi-objective function simultaneously.

In conclusion, the result indicates that the smaller of required sample sizes (lower δ) provides the optimal plan parameters of the proposed DSP_{ZINB} to achieve the maximum value of P_a, and the minimum value of *TC* and *ASN*. Based on the same value of *k*, δ, *AQL* and *LTPD*, P_a increase but *TC* and *ASN* decrease when ϕ increases. Furthermore, under three different scenarios of the required sample sizes, the S2 ($n_1 < n_2$) provides the most optimal plan parameters to achieve the optimal multi-objective. Although the S2 gives the best answer for the proposed DSP_{ZINB}, it is found that the value of the acceptance number c_1 is very different from c_2, which is not appropriate in practice. Moreover, It is seen that the proposed DSP_{ZINB} gives the optimal plan parameter when *LTPD* = 0.10. This means that the proportion of defective that will be accepted by the sampling plan at most 10% per lot. In the real case, most of the inspection processes determine that the first sample and the second sample are equal ($n_1 = n_2$). So, the performance of the proposed DSP_{ZINB} with a different value of k and ϕ are considered based on $(50,50,0,1)^*$. It can interpret that the proposed DSP_{ZINB} give a good performance when *k* is small and approaches zeros while ϕ is a large value.

To apply the proposed methods, the manufacturer should know some necessary value of input parameters such as lot size, the proportion of defect per lot, cost per unit, etc. In future work, the proposed method will be applied to construct the optimal plans of other sampling plans such as multiple acceptance sampling plans, repeat sampling plans, etc. Moreover, the proposed method can be extended under the other optimal distribution.

Abbreviations

ASP acceptance sampling plan
DSP double acceptance sampling plan
SSP single sampling plan
NB Negative Binomial
ZI Zero-inflated
ZIP Zero-inflated Poisson
ZINB Zero-inflated Negative Binomial
DSP_{ZINB} double acceptance sampling plan under the distribution
ASN average number of samples
TC total cost of inspection a lot
GA Genetic Algorithm

References

[1] D. C. Montgomery, *Introduction to Statistical Quality Control*. New York: John Wiley & Sons, 2005, pp. 681–682.
[2] M. Amitava, *Fundamentals of Quality Control and Improvement*. 4th ed., New Jersey: John Wiley & Sons, 2016, pp. 509–516.
[3] B. P. M. Duarte and P. M. Saraiva, "An optimization-based approach for designing attribute acceptance

sampling plans," *International Journal of Quality & Reliability Management*, vol. 25, pp. 824–841, 2008.

[4] I. Kaya, "A genetic algorithm approach to determine the sample size for attribute control charts," *Inform Sciences*, vol. 179, pp. 1552–1566, 2009.

[5] A. Kobilinsky and Y. Bertheau, "Minimum cost acceptance sampling plans for grain control, with application to GMO detection," *Chemometrics and Intelligent Laboratory Systems*, vol. 75, no. 2, pp. 189–200, 2005.

[6] T. M. Cheng and Y. L. Chen, "A GA mechanism for optimizing the design of attribute-double-sampling-plan," *Automation in Construction*, vol.16, no. 3, pp. 345–353, 2006.

[7] S. Sampath and S. P. Deepa, "Determination of optimal double sampling plan using genetic algorithm," *Pakistan Journal of Statistics and Operation Research*, vol. 8, no. 2, pp.195–203, 2012.

[8] O. J. Braimah, Y. K. Saheed, R.O. Owonipa, and I.O.Adegbite, "Economic reliability acceptance sampling plan design with zero acceptance," *African Journal of Computing & ICT*, vol. 8, no. 3, pp. 69–84, 2015.

[9] L. F. Hsu and J. T. Hsu, "Economic design of acceptance sampling plans in a two-stage supply chain," *Advances in Decision Sciences*, vol. 2012, pp. 1–14, 2012.

[10] M. S. FallahNezhad and M. Aslam, "A new economical design of acceptance sampling models using Bayesian inference," *Accreditation and Quality Assurance*, vol.18, no. 3, pp. 187–195, 2013.

[11] M. S. FallahNezhad and E. Qazvini, "A new economical scheme of acceptance sampling plan in a two-stage approach based on the maxima nomination sampling technique," *Transactions of the Institute of Measurement and Control*, vol. 39, pp. 1097–1103, 2017.

[12] M. S. Fallahnezhad, E. Qazvini, and M. Abessi, "Designing an economical acceptance sampling plan in the presence of inspection errors based on maxima nomination sampling method," *Scientia Iranica*, vol. 25, no. 3, pp. 1701–1711, 2018.

[13] A. Loganathan and K. Shalini, "Determination of single sampling plans by attributes under the conditions of zero-inflated Poisson distribution," *Communication in Statistics-Simulation and Computation*, vol. 43, pp. 538–548, 2014.

[14] G. Uma and K. Ramya, "Determination of quick switching system by attributes under the conditions of zero-inflated Poisson distribution," *International Journal of Statistics and Systems*, vol. 11, pp. 157–165, 2016.

[15] G. S. Rao and M. Aslam, "Resubmitted lots with single sampling plans by attributes under the conditions of zero-inflated Poisson distribution," *Communication in Statistics-Simulation and Computation*, vol. 46, pp. 1814–1824, 2017.

[16] F. K. Wang and S. S. Hailemariam, "Sampling plans for the zero-inflated Poisson distribution in the food industry," *Food Control*, vol. 85, pp. 359–368, 2018.

[17] M. Ridout, J. Hinde, and C. G. DemeAtrio, "A score test for testing a zero-inflated Poisson regression model against zero-inflated negative binomial alternatives," *Biometrics*, vol. 57, no. 1, pp. 219–223, 2001.

[18] R. Fang, "Zero-inflated negative binomial (ZINB) regression model for over-dispersed count data with excess zeros and repeated measures, an application to human microbiota sequence data," M.S. thesis, Faculty of the Graduate School, University of Colorado in partial fulfillment, 2013.

[19] M. Arianna, G. B. Ursula, and B. Francis, "Effectiveness of sampling plans by attributes based on mixture distributions characterising microbial clustering in food," *Food Control*, vol. 34, no. 1, pp. 50–60, 2013.

[20] F. K. Wang and S. S. Hailemariam, "Sampling plans for the zero-inflated negative binomial distribution in the food industry," *Quality and Reliability Engineering*, vol. 34, pp. 1174–1184, 2018.

[21] The Math WorksTM, 9.7.0 (R2019b), License Number 40791029.