Expert Systems with Applications 38 (2011) 5178–5186

Contents lists available at ScienceDirect

Expert Systems with Applications

journal homepage: www.elsevier.com/locate/eswa

Minimizing the cost of integrated systems approach to process control and maintenance model by EWMA control chart using genetic algorithm

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ARTICLE INFO

Keywords:
Integrated model
Maintenance Management
EWMA control chart
Genetic algorithm

ABSTRACT

This paper studies integrated systems approach to Statistical Process Control (SPC) and Maintenance Management (MM). Previously, only four policies which are in control alert signal, out of control alert signal, in control no signal, and out of control no signal, were used in the consideration (Zhou & Zhu, 2008). The objectives of this research are to develop an integrated model between Statistical Process Control and Planned Maintenance of the EWMA control chart. To do this, warning limit is considered to increase the policy from four to six such as warning limit alert signal and warning limit no signal. A mathematical model is given to analyze the cost of the integrated model before the genetic algorithm approach is used to find the optimal values of six variables (n,h,w,k,η,r) that minimize the hourly cost. A comparison between four-policy and six-policy models shows that the six policy model contains the hourly cost higher than that of the four policy model, it is because the addition of the warning limit in the model leads into increased ability of defective product detection. This consequently results to the increase of repairing and maintenance of machines; therefore the hourly cost is higher. Finally, multiple regressions are employed to demonstrate the effect of cost parameters.

1. Introduction

Nowadays, control charts are widely used to maintain and establish statistical control of a process. Control chart technique is well-known as a key step in production process monitoring. The control chart has major function in detecting the happening of assignable causes, so that the necessary correction could take action before non-conforming products are manufactured in a large amount. The control chart technique may be considered as both the graphical expression and operation of statistical hypothesis testing. It is recommended that if a control chart is employed to monitor process, some test parameters should be determined such as the sample size, the sampling interval between successive samples, and the control limits or critical regions of the chart.

Statistical Process Control (SPC) is an efficient technique for improvement of a firm's quality and productivity. The main objective of SPC is, similar to that of the control chart technique, to rapidly examine the occurrence of assignable causes or process shifts, and investigation of the process and corrective action should be undertaken prior to large numbers of non-conforming unit production. The SPC has two main tools in controlling the process, the "acceptance sampling" and the "control charts". The control charts are on-line process control techniques, popularly used in the process monitoring. By using control charts and collecting few but frequent samples, the SPC can function effectively to investigate changes in the process that may have affect to the product quality. One example is the EWMA control chart which is used to monitor quality characteristics of raw materials or products in a continuous process or continuous flow processes such as a chemical plant. The factory will collect data periodically on the results of analysis for determination of the percentages of certain chemical constituents.

The SPC usage is mainly to establish and maintain a state of statistical control, and identify special causes of variation. Woodall (2000) stated that differences in opinion about the purpose and scope of SPC strategy are partly because of various working in quality field, which includes quality gurus and their followers, consultants, quality engineers, industrial engineers, professional practitioners, statisticians, managers, and others. In this section, the overall purposes and scopes of SPC strategy are reviewed. During 1920s, Dr. Walter A. Shewhart and his colleagues developed the Shewhart control charts at Bell Telephone Laboratories. In 1931 defined maximum control as "condition reached when the chance cause fluctuations in a phenomenon produced by constant system of large number of chance causes in which no cause produces a predominating effect" (Shewhart, 1931). Two terms frequently mentioned in SPC are common cause and special cause variations. Shewhart advised that the distinguishing between two types of variations is primarily important for SPC in preventing over reaction or under reaction to the process. He considered common cause

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0957-4174/$ - see front matter
doi:10.1016/j.eswa.2010.10.044

Nomenclature

Cycle Time (E[T])

- T_0 the expected time searching for a false alarm
- T_P the expected time to identify maintenance requirements and to perform a Planned Maintenance
- T_A the expected time to determine occurrence of assignable causes
- T_R the expected time to identify maintenance requirements and to perform a Reactive Maintenance
- T_C the expected time to perform a Compensatory Maintenance
- τ the mean elapse time from the last sample before the assignable cause to the occurrence of the assignable cause
- ARL_O the average runs length during out-of-control period
- E the expected time to sample and chart one item
- $\gamma_P(\gamma_R, \gamma_C, \gamma_A, \gamma_w)$ the indicator variable which equals 1 if production continues during Planned Maintenance (Reactive Maintenance, Compensatory Maintenance, validate assignable cause, warning period maintenance) or 0 otherwise
- p_i^I the probability that run length of control chart equals $i$during in-control period $p_i^I = \alpha(1-\alpha)^{i-1}$
- p_i^O the probability that run length of control chart equals $i$during out-of-control period $p_i^O = (1-\beta)\beta^{i-1}$
- p_i^w the probability that run length of control chart equals $i$during warning period $p_i^w = (1-\phi(w)+\phi(-w))(\phi(w)-\phi(-w))^{i-1}$
- L the width of control limit in units of standard deviation
- d the interval between sampling

Cycle Cost (E[C])

- C_I the cost of quality loss per unit time (the process is in an in-control state) often estimated by a Taguchi Loss function
- C_O the cost of quality loss per unit time (the process is in an out-of-control state) often estimated by a Taguchi Loss function
- C_P the cost of performing Planned Maintenance
- C_R the cost of performing Reactive Maintenance
- C_C the cost of performing Compensatory Maintenance
- C_F the fixed cost of sampling
- C_V the variable cost of sampling
- C_f the cost to investigate a false alarm

Optimal Variable

- n the sampling size (n^* for optimal)
- h the interval between sampling (h^* for optimal)
- k the number of sample taken before Planned Maintenance (k^* for optimal)
- w the width of warning control limit in units of standard deviation (w^* for optimal)
- η the number of subintervals between two consecutive sampling times subintervals of length$d$, where $d = \frac{h}{\eta}$ (η^* for optimal)
- r the exponential weight constant (r^* for optimal)

of variation as set of causes attributable to inherent nature of the process that cannot be altered without changing the process itself, and for assignable cause of variation, defined as unusual shocks and disruptions to process the causes of which can and should be removed (Shewhart, 1931). Furthermore, Woodall (2000) suggested that control charts can also be used in checking process stability, he described that a process is said to be in state of "statistical control" if the probability distribution representing the quality characteristic is constant over time. On the other hand, if there are some changes in the distribution, the process is said to be "out of control". Thus the decision of acceptance/rejection according to the charted statistical value and decision regions must be made. Some authors including Juran (1998) believed that control chart and test of hypothesis have very closed relationship. However, Deming saw possibility of long term process improvement as being far more important than detection of changes. He clearly stated that meeting specification limits is not sufficient to ensure good quality, and the variability of quality characteristic should be reduced such that "specifications are lost beyond horizon" (Deming, 1986). Thus, his goal of statistical process control mostly associates with focusing the quality characteristic at the target and continuously reducing variability. For this reason, Deming remarkably supported the use of control charts but disagreed with hypothesis testing. The other opinion on the SPC came from Montgomery (1991). He stated that SPC is a powerful tools of problem solving data collection, and useful in achieving process stability and improving capability through reduction of variability. Accordingly, the principle of control chart utilization is to reduce process variability, to monitor and keep surveillance of a process, and finally to estimate product and process parameters. Of which, Montgomery believed that most important use of control chart is process improvement by reducing variability. Three main uses of control charts have been categorized by Steiner and Mackay (2000), as bellowed:

(1) To reduce the variation in an output characteristic by establishing a control chart to signal the change of an unidentified process input. The occurrence of the signal sets effort to identify this input.
(2) To determine by when and by how much a process should be adjusted. A control chart is setup and adjustments are made only when a signal occurs.
(3) To demonstrate process being stable and capable. The purpose here is to provide information to make decision regarding the receiving inspection.

Box and Luceno (1997) discussed that in order to obtain successful implementation of a control chart, practitioner requires to make three important decisions: (1) Is control chart an appropriate tool for application?, (2) Which type of control chart should be used?, and (3) Where should control limits be placed?. The authors further commented that for the first question, the answer depends on whether or not stable periods without changes in process mean or variance exist. If there is a stable variance but the process mean drifts, then automatic process control strategy should be considered as a means of reducing variability. In case of the second and third questions, the answers will depend on the purpose on the use of these charts, i.e., detection of real time process monitoring, problem solving, assessment of process stability, and nature of disturbance.

Extensive utilization of control charts have been seen in monitoring process stability and capability. Function of control charts are based on representing data or quality-related characteristics of the product or service. For instance, variable control charts are mostly used for measurable characteristics on numerical scales. In case of the quality-related characteristics, this usually cannot be easily represented in numerical form so attribute control charts may be useful (Gulbay & Kahraman, 2006). Generally, monitoring and determination of process are concentrated on the "under control" or "out of control" process To perform this, other quality constraints like quality cost, rate of errors, acceptance probability, consumer and producer risks, etc. must be accounted in the consideration. Lorenzen and Vance (1986) proposed a general method for determining the economic design of control charts. The advantage of this method is its application regardless of the statistic used. It is necessary to calculate only the average run-length of the statistics when assuming that the process is in-control and assuming that the process is out-of-control in some specified manner. Alexander, Dillman, Usher, and Damodaran (1995) developed a loss model for estimating the three parameters from combination between the Duncan's cost model and the Taguchi Loss function. This loss model explicitly considers the quality. Rahim and Banerjee (1993) determined jointly the optimal design parameters on a $\overline{X}$ control chart and preventive maintenance (PM) time for a production system with an increasing failure rate. Other aspects on economic design of control charts have also been discussed. Development on the economic design of control charts for monitoring the process mean has been extensively investigated in the literatures of Montgomery (1980), and Ho and Case (1994a). Rahim (1994), Ben-Daya (1999), Ben-Daya and Rahim (2000) investigated integration of $\overline{X}$ chart and PM for using in the deteriorating process of in-control period follows a general probability distribution with increasing hazard rate. Pongpullponsak et al. (2009) studied a $\overline{X}$ chart in conjunction with an age replacement preventive maintenance policy. Ben-Daya and Rahim (2001) provided an overview of the literature dealing with integrated models for production, schedule, quality control and maintenance policy. Recently, Pongpullponsak et al. (2009) introduced an economic model and using Shewhart method to compare the efficiency of $\overline{X}$ control chart for skewed distributions. The results indicated that the production level begins to vary from 3.0 s of lognormal distribution. The lowest expense was observed at the coefficient of skewness at (α_3) 6. Panagiotidou and Tagaras (2007) analyzed an economic model for the optimization of preventive maintenance in a production process with two quality states. Ho and Case (1994b) presented a literature on control charts employing an EWMA type statistic, while several authors (e.g., Chou, Cheng, & Lai, 2008; Ho & Case, 1994b; Torng, Montgomery, & Cochran, 1994) have explored the economic design of EWMA control charts to monitor the process mean. Park, Lee, and Kim (2004) extended the traditional economic design of an EWMA chart to the case where the sampling interval and sample size may vary depending on the current chart statistic. Park and Reynolds (2008) considered IPC monitoring schemes using an economic design approach under the inherent wandered of the process. Subsequently, it can be represented as an ARIMA (0,1,1) model. In the model, they considered a combination of two EWMA charts, with one EWMA statistic using the observed deviations from the target, and the other EWMA statistic using the squared deviations from the target. It was found that, if only one control chart is planed to be used for simplicity, then the two EWMA control chart provides very good performance and this causes the chart more preferable to be used than others. However, it should be noted that the EWMA chart is the standard. Although the control chart is considered for monitoring a process in the current setting, but the two EWMA control chart, actually have much better performances. Serel (2009) studied the case where the assignable cause changes only the process mean or dispersion. The economic design of EWMA mean charts was extended to the case where the quality related costs are computed based on a loss function. Serel and Moskowitz (2008) showed that when the assignable causes lead to changes in both process mean and variance, simultaneous use of mean and dispersion charts is important for detecting the changes quickly. In their work, joint economic design of EWMA charts for process mean and dispersion have been explored.

The aim of this work is to develop the integrated economic design of EWMA control chart for determining the values of six test variables of the chart (which are the sample size (n), the sampling interval (h), the number of subintervals between two consecutive sampling times (η), the warning limit coefficient (w), the number of sample taken before planned maintenance (k), and the exponential weight constant (r)). By using this developed genetic algorithm to optimize these parameters (six test parameters), the total cost per hour ($E[H]$) is expectedly minimized.

2. Model consideration

In this work, we develop an integrated model of control chart with reference to the three-scenario integrated model firstly purposed by Linderman, McKone-Sweet, and Anderson (2005). Then a generalized analytic model is employed to determine the optimal policy for using coordinated with Statistical Process Control and Planned Maintenance in minimizing the total expected cost. Recently, Zhou and Zhu (2008) modified the Linderman model from three to four policies under determination of the optimal policy for minimization of the total expected cost with coordination of Statistical Process Control and Planned Maintenance. For this research, an integrated model between Statistical Process Control and Planned Maintenance of the EWMA control chart is conducted. In developing, warning limit is considered to increase policy from four policies to six policies such as warning limit alert signal and warning limit no signal. As shown in Fig. 1, the framework of the integrated model illustrates six different scenarios, in which each scenario is further elaborated as following. In Scenario 1, the process begins with a "in-control" state and inspections occur after h hours of monitoring as to whether the process has shifted from an "in-control" to an "out-of-control" state. There is an alert signal in the control chart before the scheduled time when maintenance should be performed. But the signal is false, that is to say, the process is still "in-control". Since searching and determining false signal take time and incur cost, Compensatory Maintenance is performed. In Scenario 2, similar to Scenario 1, there is also a signal. While the signal is valid and the process shifts to an "out-of-control" state, it results in Reactive Maintenance. In Scenario 3, the process begins with a "in-control" state and inspections occur after h hours of monitoring as to whether the process has shifted from an "in-control" to an "warning limit" state. And there is an alert signal in the control chart before the scheduled time when maintenance should be performed. In Scenarios 4 and 5, no signal occurs in the control chart before the scheduled time. Then at the $(k+1)$th sampling interval, appropriate maintenance should be arranged. In Scenario 4, the process is always "in-control", we perform Planned Maintenance. When the process shifts to an

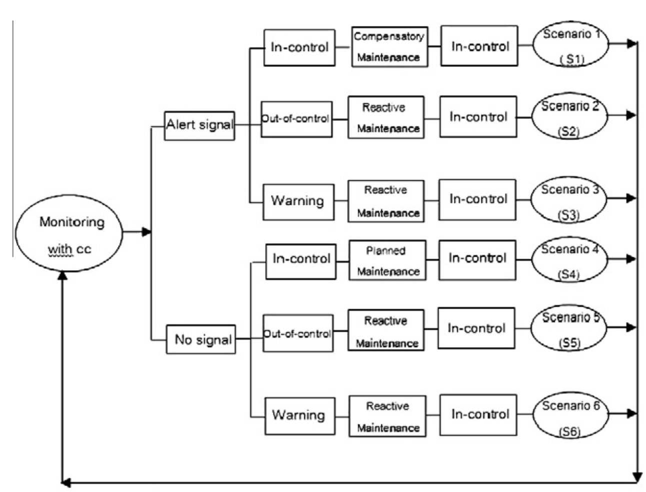

Fig. 1. Six monitoring – maintenance scenarios of the integrated model.

"out-of-control" state in Scenario 5, Reactive Maintenance takes place because the "out-of-control" condition occurred before the scheduled time, and additional time and expense will be incurred to identify and solve the equipment problem. In Scenario 6, the process begins with an "in-control" state and no signal occurs in the control chart before the scheduled time. Then at the $(k+1)$th sampling interval, appropriate maintenance should be arranged. And in Scenario 6, the process is always "in-control", we perform Planned Maintenance.

3. Cost analysis of integrated model

The process begins with an in-control state with a Process Failure Mechanism that follows a Weibull distribution. Denote,

$$f(t)=\lambda^{v}vt^{v-1}e^{-(\lambda t)^{v}}\quad \text{where}\quad \lambda, v, t\geqslant 0$$

The Weibull cumulative distribution function has the form as

$$F(t)=1-e^{(\lambda t)^{v}}\quad \text{where}\quad \lambda, v, t\geqslant 0$$

At this stage, we develop the integrated model based on the Linderman model (2005) and the general cost function of Lorenzen and Vance (1986), and assumed six models of the integrated system are considered which are Scenarios 1–6. After that, the expected cycle time and cycle cost for each of the six scenarios are investigated as a following:

Scenario 1 (*S1*). The process begins with a "in-control" state and inspections occur after h hours of monitoring as to whether the process has shifted from an "in-control" to an "out-of-control" state. And there is an alert signal in the control chart before the scheduled time when maintenance should be performed. But the signal is false, that is to say, the process is still "in-control". Since searching and determining false signal take time and incur cost, Compensatory Maintenance is performed.

$$E[T|S_1]=h\sum_{i=1}^{k}ip_i^{I}(1-F(ih))+T_0+T_C$$

$$E[C|S_1]=C_I\left[h\sum_{i=0}^{k}ip_i^{I}(1-F(ih))+\gamma_C T_C\right]+(C_F+nC_V)\sum_{i=0}^{k}ip_i^{I}(1-F(ih))+C_f+C_C$$

Scenario 2 (*S2*). It assumes that the process shifts to an "out-of-control" state prior to the Planned Maintenance and process failure mechanism follows a Weibull distribution, the in-control time follows a truncated Weibull distribution.

$$f(t|(k+1)h)=\frac{f(t)}{F((k+1)h)}=\frac{\lambda^{v}vt^{v-1}e^{-(\lambda t)^{v}}}{1-e^{-(\lambda(k+1)h)^{v}}};\quad 0\leqslant t\leqslant (k+1)h$$

then we have

$$E[T|S_2]=\int_0^{kh}tf(t|(k+1)h)dt+hARL_O-\tau+nE+T_A+T_R$$

where $\tau=\sum_{i=0}^{k}\int_{ih}^{(i+1)h}(t-ih)f(t|(k+1)h)dt$, ARL_O: The average run length during out-of-control period[]

$$ARL_O=\frac{1}{1-\phi\left(L-\frac{\varepsilon}{\sigma_{\bar{X}}\sqrt{\frac{r}{1-r}}}\right)+\phi\left(-L-\frac{\varepsilon}{\sigma_{\bar{X}}\sqrt{\frac{r}{1-r}}}\right)}$$

$$E[C|S_2]=C_I\left[\int_0^{kh}tf(t|(k+1)h)dt\right]+C_O[hARL_O-\tau+nE+\gamma_A T_A+\gamma_R T_R]+\frac{1}{h}E[T|S_2](C_F+nC_V)+C_R \tag{1}$$

Scenario 3 (*S3*). The process begins with a "in-control" state and inspections occur after h hours of monitoring as to whether the process has shifted from an "in-control" to an "warning limit" state. There is an alert signal in the control chart before the scheduled time when maintenance should be performed.

$$E[T|S_3]=\int_0^{khr}tf(t|(k+1)h)dt+(1-\gamma_1)sT_0\alpha+hARL_O-\xi+nE+T_A+T_R \tag{2}$$

where s is the expected sampling frequency while in control

$$s=1+\left(\frac{1-\rho^{\eta-1}}{1-\rho}+(2\eta-1)\rho^{\eta-1}\right)\left[\sum_{j=2}^{\infty}\left(1-\int_0^{j-1}\lambda^{v}vt^{v-1}e^{-(\lambda t)^{v}}dt\right)\right] \tag{3}$$

ρ is the conditional probability that the sample point is plotted in the warning region. Given that the process is in control and is equal to

$$\rho=\frac{2[\phi(L)-\phi(w)]}{\phi(L)-\phi(-L)} \tag{4}$$

γ_w is the indicator variable which equals 1 if production continues during warning period or 0 otherwise, ξ is average time lag between the sampling time point, which is just prior to the occurrence of the assignable cause, and the time point that the assignable cause occurs. And it can be shown that

$$\xi=\sum_{j=0}^{\eta-1}(p_{1j}\tau_{1j})+p_2\tau_2 \tag{5}$$

p_{1j} is the ratio of the sampling interval $h-jd$ to the average sampling interval and is equal to

$$p_{1j}=\frac{\rho^{j}(1-\rho)h-jd}{\rho\sum_{i=0}^{\eta-2}\rho^{i}d+(1-\rho)\sum_{i=0}^{\eta-1}\rho^{i}(h-id)}\quad \text{for}\quad j=0,1,2,\ldots,\eta-1 \tag{6}$$

Suppose that the interval h between two fixed times is divided into η subintervals of length d, where $d=\frac{h}{\eta}$, p_2 is the ratio of the sampling interval d to the average sampling interval and is equal to

$$p_2=\frac{\rho\sum_{i=0}^{\eta-2}\rho^{i}d}{\rho\sum_{i=0}^{\eta-2}\rho^{i}d+(1-\rho)\sum_{i=0}^{\eta-1}\rho^{i}(h-id)} \tag{7}$$

τ_{1j} is the assignable cause occurred between the sampling time points $ih+jd$ and $(i+1)h$.

$$\tau_{10}=\frac{\int_{ih}^{(i+1)h}t^{v}e^{-(\lambda t)^{v}}dt}{\int_{ih}^{(i+1)h}t^{v-1}e^{-(\lambda t)^{v}}dt}-ih, \tag{8}$$

$$\tau_{1j}=\frac{\int_{ih+jd}^{(i+1)h}t^{v}e^{-(\lambda t)^{v}}dt}{\int_{ih+jd}^{(i+1)h}t^{v-1}e^{-(\lambda t)^{v}}dt}-ih-jd\quad \text{for}\quad j=1,2,3,\ldots,\eta-1 \tag{9}$$

τ_2 is the assignable cause occurred between the ith and $(i+1)$st sampling time points with sampling interval d. The expected in-control time interval during this period may be written as

$$\tau_2=\frac{\int_{id}^{(i+1)d}t^{v}e^{-(\lambda t)^{v}}dt}{\int_{id}^{(i+1)d}t^{v-1}e^{-(\lambda t)^{v}}dt}-id \tag{10}$$

$$E[C|S_3]=C_I\left[\int_0^{kh}tf(t|(k+1)h)dt\right]+C_O[hARL_O-\xi+nE+\gamma_A T_A+\gamma_R T_R]+\frac{1}{\rho\sum_{i=0}^{\eta-2}\rho^{i}d+(1-\rho)\sum_{i=0}^{\eta-1}\rho^{i}(h-id)}E[T|S_3](C_F+nC_V)+C_f s\alpha+C_R \tag{11}$$

Scenario 4 (*S4*). No signal occurs in the control chart before the scheduled time. Then, at the $(k+1)$th sampling interval, appropriate maintenance should be arranged. In S4, the process is always "in-control", we perform Planned Maintenance.

$$E[T|S_4] = \int_0^{kh} tf(t|(k+1)h)dt(k+1)h - \int_0^{kh} tf(t|(k+1)h)dt + T_p = (k+1)h + T_p \quad (12)$$

$$E[C|S_4] = C_I[(k+1)h + \gamma_p T_p] + k(C_F + nC_V) + C_p \quad (13)$$

Scenario 5 (*S5*). The process begins in control. When the process shifts to an "out-of-control" state, Reactive Maintenance takes place because the "out-of-control" condition occurred before the scheduled time, and additional time and expense will be incurred to identify and solve the equipment problem.

$$E[T|S_5] = (k+1)h + T_R \quad (14)$$

$$E[C|S_5] = C_I\left(\int_0^{kh} tf(t|(k+1)h)dt\right) + C_O\left[(k+1)h - \int_0^{kh} tf(t|(k+1)h)dt + \gamma_R T_R\right] + k(C_F + nC_V) + C_R \quad (15)$$

Scenario 6 (*S6*). The process begins with an "in-control" state and no signal occurs in the control chart before the scheduled time. Then at the $(k+1)$th sampling interval, appropriate maintenance should be arranged. In S6, the process is always "in-control", we perform Reactive Maintenance.

$$E[T|S_6] = (k+1)h + T_R \quad (16)$$

$$E[C|S_6] = C_I\left(\int_0^{kh} tf(t|(k+1)h)dt\right) + C_O\left[(k+1)h - \int_0^{kh} tf(t|(k+1)h)dt + \gamma_R T_R\right] + k(C_F + nC_V) + C_R \quad (17)$$

Next, determination of the hourly cost ($E[H]$) is performed.

The model can be considered as a renewal-reward process; hence, the expected cost per hour $E[H]$ can be expressed as

$$E[H] = \frac{E[C]}{E[T]} \quad (18)$$

where

$$E[T] = E[T|S_1]P(S_1) + E[T|S_2]P(S_2) + E[T|S_3]P(S_3) + E[T|S_4]P(S_4) + E[T|S_5]P(S_5) + E[T|S_6]P(S_6) \quad (19)$$

$$E[C] = E[C|S_1]P(S_1) + E[C|S_2]P(S_2) + E[C|S_3]P(S_3) + E[C|S_4]P(S_4) + E[C|S_5]P(S_5) + E[C|S_6]P(S_6) \quad (20)$$

and probability of Scenario 1

$$P(S_1) = \sum_{i=1}^{k} P(\text{In-control} \cap \text{Alert Signal}) = \sum_{i=1}^{k} P(\text{In-control}|\text{Alert Signal})P(\text{Alert Signal}) = \sum_{i=1}^{k} p_i^I(1 - F(ih)) \quad (21)$$

Probability of Scenario 2

$$P(S_2) = \sum_{i=1}^{k} P(\text{Out-of-control} \cap \text{Alert Signal}) = \sum_{i=1}^{k} P(\text{Out-of-control}|\text{Alert Signal})P(\text{Alert Signal}) = \sum_{i=1}^{k}[F(ih) - F(i-1)h]\left(1 - \sum_{j=1}^{i-1} p_j^I\right)\sum_{l=1}^{k-i+1} p_l^O \quad (22)$$

Probability of Scenario 3

$$P(S_3) = \sum_{i=1}^{k} P(\text{Warning limit} \cap \text{Alert Signal}) = \sum_{i=1}^{k} P(\text{Warning limit}|\text{Alert Signal})P(\text{Alert Signal}) = \sum_{i=1}^{k}[F(ih) - F(i-1)h]\left(1 - \sum_{j=1}^{i-1} p_j^I\right)\sum_{l=1}^{k-i+1} p_l^w \quad (23)$$

Probability of Scenario 4

$$P(S_4) = \sum_{i=1}^{k} P(\text{In-control} \cap \text{No Signal}) = \sum_{i=1}^{k} P(\text{In-control}|\text{No Signal})P(\text{No Signal}) = (1 - F(kh)) - \sum_{i=1}^{k} p_i^I(1 - F(ih)) \quad (24)$$

Probability of Scenario 5

$$P(S_5) = \sum_{i=1}^{k} P(\text{Out-of-control} \cap \text{No Signal}) = \sum_{i=1}^{k} P(\text{Out-of-ontrol}|\text{No Signal})P(\text{No Signal}) = F(kh) - \sum_{i=1}^{k}[F(ih) - F(i-1)h]\left(1 - \sum_{j=1}^{i-1} p_j^I\right)\sum_{l=1}^{k-i+1} p_l^O \quad (25)$$

Probability of Scenario 6

$$P(S_6) = \sum_{i=1}^{k} P(\text{Warning limit} \cap \text{No Signal}) = \sum_{i=1}^{k} P(\text{Warning limit}|\text{No Signal})P(\text{No Signal}) = F(kh) - \sum_{i=1}^{k}[F(ih) - F(i-1)h]\left(1 - \sum_{j=1}^{i-1} p_j^I\right)\sum_{l=1}^{k-i+1} p_l^w \quad (26)$$

The economic design of integrated model of EWMA chart is aimed to be used in determining the optimal values of the six test variables (n, h, k, w, η, r) such that the expected total cost per hour in Eq. (18) is minimized.

From examination of the components in Eqs. (19) and (20), it can be seen that determining the economically optimal values of the six test variables for the EWMA chart is not straightforward. To illustrate the nature of the solutions obtained from economic design of EWMA chart, a particular numerical example is provided.

4. A numerical example and solution procedure

The solution procedure is carried out using genetic algorithms (GA) with MATLAB 7.6.0(R2009a) software to obtain the optimal values of n, h, k, w, η and r which will be subsequently used to minimize ($E[H]$).

The GA is a simulation computer program, evolved from the concept of natural genetics and biological evolution, that is used as a random search technique for optimization purpose. To date, the current GA originated from the models of Holland (1975) has been applied for several fields such as bioinformatics, computational science, engineering, mathematics, and manufacturing. For the GA theory, the solution of a problem is called a "chromosome". Naturally, a chromosome is composed of a number of genes (the genetics materials controlled features or characters of individuals). Compared to other kinds of numerical optimization methods, such as neural network, gradient-based search, etc., the GA has promising points in the following aspects:

1. In GA operation, the fitness function values and the stochastic way (not deterministic rule) are employed to seek for the search direction of the optimal solution optimization. For this reason, the GA is capable of being applied for many kinds of optimization problems.
2. The ability of GA in a global optimum by mutation and crossover facilitates avoidance of being trapped in the local optimum. In the other words, it can serve as a promising approach to solve complex problems.
3. Since the GA is able to search for many possible solutions (or chromosomes) within one operation, the global optimal solution can be achieved efficiently.

From the outstanding advantages discussed above, the GA extensively serves as a tool for solving the problems of combinatorial optimization in many areas (e.g., Chou, Chen, & Chen, 2006; Chou, Wu, & Chen, 2006; Jensen, 2003). For our example, the solution process using the GA by MATLAB 7.6.0(R2009a) is briefly described as follows:

Step 1. *Initialization:* The procedure starts at randomly generating 100 solutions that reach the constraint condition of individual test parameter. Meanwhile, the constraint condition represented for individual test parameter is set as bellows:

$$1 \leqslant n \leqslant 25,\ 0.1 \leqslant h \leqslant 5,\ 2 \leqslant \eta \leqslant 10,\ 2 \leqslant w \leqslant 2.5,$$
$$20 \leqslant k \leqslant 40,\ 0.1 \leqslant r \leqslant 0.2$$

Step 2. *Evaluation:* This step is to define the fitness of individual solution by calculating the value of fitness function. For this research, the fitness function is the cost function shown in Eq. (11).

Step 3. *Selection:* After obtained the fitness expression, 30 solutions that are likely to be better fitness of solutions are selected for breeding the next generation. (For the first generation the chromosome with the lowest cost is selected to replace the highest cost chromosome.)

Step 4. *Crossover:* To produce the new chromosomes for the next generation, a pairs of parent solutions (from the 30 solutions) are selected randomly and used for crossover operations. In this example, we apply the arithmetical crossover method with crossover rate 0.8 as follows:

$$D_1 = 0.8R + 0.2M,\ D_2 = 0.2R + 0.8M$$

where D_1 is the first new chromosome, D_2 is the second new chromosome, and R and M are the parents chromosomes. Thus, if 30 randomly selected parents are used, then there will be 60 children are produced. Now, we have the population size of 90 solutions (i.e., 30 parents + 60 children) for next generation.

Step 5. *Mutation:* Suppose that the mutation rate is 0.1. In this illustration, we use non-uniform method to carry out the mutation operation. As we have 90 solutions, nine chromosomes (i.e., $90 \times 0.1 = 9$) can be randomly selected to mutate some parameters (or genes).

Step 6. Repeat Step 2 to Step 5 until the population reaches the stopping criteria. For our experiment, the procedure is repeated until no more value to change which is our stopping criteria.

In this section, we test the effect of model parameters on the solution of economic design of the EWMA chart by conducting numerical example and sensitivity analysis. For the sensitivity determination, orthogonal-array experimental design and multiple regression is used. In the analysis performance, the model parameters are fixed as the independent variables, while the six test parameters (i.e., n, h, k, w, η and r), and the average total hourly cost $E[H]$, are set as the dependent variables. In Table 1, eight independent parameters (i.e., the model parameters) to be tested in the sensitivity analysis and their corresponding level planning are illustrated.

The experiment is carried out using the L_{16} orthogonal array. As shown in Table 2, the eight independent parameters are then assigned to the columns of the L_{16}array. In the L_{16} orthogonal array experimental design, there are 16 trials (i.e., 16 different level combinations of the independent variables). For each trial, the GA is applied to produce the optimal solution of the economic design, with the following model parameters fixed:

$$\gamma_1 = \gamma_2 = \gamma_3 = 1,\ \lambda = 0.05,\ L = 3,\ \nu = 2,\ T_0 = 0.2,\ T_C = 0.6,$$
$$T_A = 0.3,\ T_R = 1,\ T_P = 0.8$$

The best value parameters by L_{16} orthogonal array are show in Table 3.

The best values of 6 variables estimated by the GA are shown in Table 4.

From Table 4, the optimal values of the policy variables those minimize $E[H]$ are $n^* = 6.082$, $h^* = 3.008$, $w^* = 2.494$, $\eta^* = 5.006$, $r^* = 0.2$, and the corresponding hourly cost is $E[H] = 194.640$.

The outputs of the GA obtained from every trial is shown in Table 2. The next step is to analyze the effect of model parameters on the solution of economic design of EWMA chart. To do this the data of each dependent variable in Table 2 is run the regression analysis by the statistical software (SPSS 15.0). It is found that the output of SPSS covered an ANOVA regression table and a regression table for each dependent variable, shows the information corresponding to statistical hypothesis testing.

Demonstrating in Table 5 is the SPSS output for the sample size ($\hat{n}$). Considered the ANOVA in Table 5(a), if the significance level is set to be 0.05, we observe that at least one model parameters significantly affect the value of sample size ($\hat{n}$).

By examining Table 5(b), we find that the cost of quality loss per unit time (the process is in an in-control state) often estimated by a Taguchi Loss function (C_I) significantly affect the value of sample size ($\hat{n}$). It is noticed that the sign of the coefficients of the cost

Table 1
Eight model parameters and their level planning.

Model parameter	Level 1	Level 2
C_I	10	20
C_O	200	400
C_P	3000	6000
C_R	2000	4000
C_C	1000	2000
C_F	10	20
C_V	0.1	0.2
C_f	100	200

Table 2
Model parameter assignment in the L_{16} orthogonal array and the corresponding solution.

Trial	Model parameter							
	C_I	C_O	C_P	C_R	C_C	C_F	C_V	C_f
1	10	200	3000	2000	1000	10	0.1	100
2	20	200	3000	2000	1000	20	0.2	200
3	10	400	3000	2000	2000	10	0.2	200
4	20	400	3000	2000	2000	20	0.1	100
5	20	200	6000	2000	2000	20	0.2	100
6	10	200	6000	2000	2000	10	0.1	200
7	20	400	6000	2000	1000	20	0.1	200
8	10	400	6000	2000	1000	10	0.2	100
9	20	200	3000	4000	2000	20	0.1	200
10	10	200	3000	4000	2000	10	0.2	100
11	20	400	3000	4000	1000	20	0.2	100
12	10	400	3000	4000	1000	10	0.1	200
13	10	200	6000	4000	1000	10	0.2	200
14	20	200	6000	4000	1000	20	0.1	100
15	10	400	6000	4000	2000	10	0.1	100
16	20	400	6000	4000	2000	20	0.2	200

	Solution						
	n	h	k	w	η	r	$E[H]$
1	5.046	3.002	39.13	2.468	5.004	0.19	194.8
2	5	3	20	2.5	6.641	0.2	196.437
3	5	3	20	2.499	6.618	0.2	373.79
4	6.803	3	20.16	2.498	8.043	0.197	377.055
5	5.034	3.006	20.28	2.497	5.003	0.2	195.46
6	6.082	3.008	20.66	2.494	5.006	0.2	194.64
7	5.045	3.002	20	2.5	5.009	0.2	374.881
8	5.146	3.01	20.01	2.493	5.007	0.2	374.141
9	5.634	4.994	39.08	2.03	5.014	0.01	201.744
10	5.068	4.997	20.06	2.497	5.038	0.042	201.796
11	5	3	20	2.5	6.183	0.2	385.722
12	5.791	3	20.04	2.499	5	0.2	384.504
13	5.009	4.999	20.04	2.5	5.045	0.043	201.477
14	6.303	4.998	20.49	2.496	5.011	0.042	202.231
15	5	3	24.34	2.5	5	0.2	385.642
16	6.557	3.001	20.14	2.5	5.058	0.2	386.679

Table 3
Optimal values for model parameters.

Parameter	Value	Parameter	Value	Parameter	Value
C_I	20	C_R	2000	C_V	0.1
C_O	200	C_C	2000	C_f	200
C_P	6000	C_F	10		

Table 4
Optimal values for six variables and the optimal value of the total hourly costs.

Variable	Integrated model (EWMA)
n^*	6.082
h^*	3.008
k^*	20.66
w^*	2.494
η^*	5.006
r^*	0.2
$E[H]$	194.640

of quality loss per unit time (the process is in an in-control state) often estimated by a Taguchi Loss function (C_I) is positive, indicating that the higher cost of quality loss per unit time (the process is in an in-control state) often estimated by a Taguchi Loss function (C_I) generally increases the sample size ($\hat{n}$), which is consistent with the principle of statistical hypothesis testing.

Table 6 is the SPSS output for the interval between sampling ($\hat{h}$). It can be seen from the ANOVA in Table 6(a) that if the significance level is set to be 0.05, then there are at least one model parameters that significantly affect the value of the interval between sampling ($\hat{h}$).

By examining Table 6(b), we find that the cost of quality loss per unit time (the process is in an out-of-control state) often estimated by a Taguchi Loss function (C_O) significantly affect the value of the interval between sampling ($\hat{h}$). It is noticed that the sign of the coefficients of the cost of quality loss per unit time (the process is in an out-of-control state) often estimated by a Taguchi Loss function (C_O) is negative, indicating that the higher cost of quality loss per unit time (the process is in an out-of-control state) often estimated by a Taguchi Loss function (C_O) generally reduces the interval between sampling ($\hat{h}$), which is consistent with the principle of statistical hypothesis testing.

Table 7 is the SPSS output for the exponential weight constant ($\hat{r}$). For the ANOVA in Table 7(a), if the significance level is set at 0.05, then at least one model parameters significantly affect the value of the exponential weight constant ($\hat{r}$).

Table 5
SPSS output for the sample size ($\hat{n}$).

(a) ANOVA Table

Model	Sum of squares	df	Mean square	F	P-value
Regression	1.658	1	1.658	5.744	0.031(a)
Residual	4.042	14	0.289		
Total	5.7	15			

(b) Table of regression coefficients

Independent variable	Coefficient	Std. error	t	P-Value
(Constant)	4.452	0.425	10.481**	0.00
C_I	0.064	0.027	2.397*	0.031

(a) Predictors: (Constant), C_I.

Table 6
SPSS output for the interval between sampling ($\hat{h}$).

(a) ANOVA Table

Model	Sum of squares	df	Mean square	F	P-Value
Regression	7.938	2	3.969	12.941	0.001(a)
Residual	3.987	13	0.307		
Total	11.925	15			

(b) Table of regression coefficients

Independent variable	Coefficient	Std. error	t	P-Value
(Constant)	3.5	0.603	5.8**	0.00
C_O	−0.005	0.001	−3.958**	0.003

(a) Predictors: (Constant), C_O.

Table 7
SPSS output for the exponential weight constant ($\hat{r}$).

(a) ANOVA Table

Model	Sum of squares	df	Mean square	F	P-Value
Regression	0.054	2	0.027	12.751	0.001(a)
Residual	0.028	13	0.002		
Total	0.082	15			

(b) Table of regression coefficients

Independent variable	Coefficient	Std. error	t	P-Value
(Constant)	0.154	0.05	3.058**	0.009
C_R	−3.9	0	−3.516**	0.004

(a) Predictors: (Constant), C_R.

Table 8
SPSS output for the Hourly Cost ($\widehat{E[H]}$).

(a) ANOVA Table

Model	Sum of squares	df	Mean square	F	P-Value
Regression	132407.8	2	66203.889	30055.7	0.00(*a*)
Residual	28.635	13	2.203		
Total	132436.4	15			

(b) Table of regression coefficients

Independent variable	Coefficient	Std. error	*t*	P-Value
(Constant)	3.955	1.617	2.445*	0.029
C_O	0.909	0.004	224.903**	0.00
C_R	0.004	0.000	11.565**	0.00

(a) Predictors: (Constant), C_O, C_R.

Considering Table 7(b), it is found that the coefficients of the cost of performing Reactive Maintenance (C_R) significantly affect the value of the exponential weight constant ($\hat{r}$). It is noticed that the sign of the coefficients of the cost of performing Reactive Maintenance (C_R) is negative, indicating that the higher cost of performing Reactive Maintenance (C_R) generally reduces the exponential weight constant ($\hat{r}$), which is consistent with the principle of statistical hypothesis testing.

Table 8 is the SPSS output for the Hourly Cost ($E[\hat{H}]$). It can be seen from the ANOVA in Table 8(a) that if the significance level is set to be 0.05, then there are at least one model parameters that significantly affect the value of the Hourly Cost ($E[\hat{H}]$).

By examining Table 8(b), we find that the cost of quality loss per unit time (the process is in an out-of-control state) often estimated by a Taguchi Loss function (C_O) and the cost of performing Reactive Maintenance (C_R) significantly affect the value of the Hourly Cost ($E[\hat{H}]$). It is noticed that the sign of the coefficients of the cost of quality loss per unit time (the process is in an out-of-control state) often estimated by a Taguchi Loss function (C_O) is positive, indicating that the higher cost of quality loss per unit time (the process is in an out-of-control state) often estimated by a Taguchi Loss function (C_O) generally increases the Hourly Cost ($E[\hat{H}]$). And the sign of the coefficients of the cost of performing Reactive Maintenance (C_R) is positive, indicating that the higher cost of performing Reactive Maintenance (C_R) generally increases the Hourly Cost ($E[\hat{H}]$), which is consistent with the principle of statistical hypothesis testing.

5. Conclusions

In the present paper, we present an integrated model which is based on two classical manufacturing process control tools, Statistical Process Control and Maintenance Management. The development of the integrated economic design of EWMA control chart is on the purpose of utilization for determining the values of six test variables of the chart (i.e., the sample size (n), the sampling interval (h), the number of subintervals between two consecutive sampling times (η), the warning limit coefficient (w), the number of samples taken before Planned Maintenance, (k) and the exponential weight constant (r)), such that the expected total cost per hour ($E[H]$) is expectedly minimized. Establishment of the cost function is based on the cost model described in Linderman et al. (2005) with demonstration of an illustrative example. After using the GA in search for the solution of the economic design, a sensitivity analysis was then performed in order to test the effect of model parameters on the solution of the economic design. Observation of the results from the sensitivity analysis, revealed that:

1. A higher cost of quality loss per unit time (the process is in an in-control state) often estimated by a Taguchi Loss function (C_I) generally results in the increased sample size (n).
2. A higher cost of quality loss per unit time (the process is in an out-of-control state) often estimated by a Taguchi Loss function (C_O) generally leads to the reduction of the interval between sampling (h).
3. A higher cost of performing Reactive Maintenance (C_R) generally results to the reduced exponential weight constant (r).
4. A higher cost of quality loss per unit time (the process is in an out-of-control state) often estimated by a Taguchi Loss function (C_O) generally results in the increase of the Hourly Cost ($E[H]$).
5. A higher cost of performing Reactive Maintenance (C_R) generally results to the increase of the Hourly Cost ($E[H]$).

6. Suggestion

Using the GA in this research, we fulfill our purpose in obtaining the optimal variables and minimum hourly cost. However, we have some suggestion as following. Increasing the policy from three policies and four policies to six policies affects to the hourly cost value, that is, apart from the increased number of the quality product the hourly costs are increasing. Basically, the sampling is increasing when the warning limit is used to check for low-quality or nonconforming product. Using our integrated model, the optimal warning limit (w) that minimizes the hourly cost is $w = 2.49$, while the value of warning limit (w) commonly used is 2.00. Besides, the general value of exponential weight (r) is 0.1, while the value of exponential weight (r) in this research is 0.2. As the hourly cost value depends on $C_O$andC_R, if the number of non-conforming products is increased the overall hourly cost will be increased too. Essentially, although our optimal warning limit (w)(2.49) is higher than that of the common value (2.00), less non-conforming product manufacturing occurs, subsequently the value of overall total hourly cost is most likely lower.

7. Future work

In addition to the GA, there still have several other potential optimization techniques, thus other optimization techniques are being tested to optimize the integrated model by EWMA control chart such as pattern search, simulated annealing algorithm, and threshold acceptance algorithm. Then, the obtained results will be compared with the result from the GA.

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