Refractive Index Finding using Multiple Light Reflections with Modified Sagnac Interferometer

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***ABSTRACT* - Refractive index finding using multiple light reflections with modified Sagnac interferometer is proposed here. The thickness of the material and fringes pattern are fundamental parameters in such measurement. In this proposed technique, multiple light reflections, by the two parallel mirrors attached to a piezo-electric transducer (PZT) vibrating at a frequency of 420Hz, introduced the phase retardation. The phase retardation and reflected angles could be found and employed for refractive index determination. Four identical interferograms were obtained by a half-wave plate (HWP) oriented at 0, π/2, π and 3π/2 rad with respect to the incident plane. The intensities at their respective patterns were evaluated and the phase shift could be computed. Using numerous reflections with a Sagnac interferometer, this technique is presented to obtain multiple fringes with a sample fixed in position at certain carrier frequencies.**

Keywords-*, *multiple-reflections, Sagnac interferometer, interferogram

I. INTRODUCTION

Optical measurements using phase-shifting technique have been in good agreement with many approaches for high accuracy, good selectivity, error-free, and high resolutions. Refractive index finding is one of the important applications using an optical interferometry. The interference fringes, formed by two beams with different path lengths when a sample is placed in the sensing arm, are the key characteristic for measuring the material's refractive index. Several research has been achieved with Mach Zehnder and Michelson interferometer to increase the number of fringes [1][2]. Michelson interferometer is a technique employing multiple reflections of light beams within the two mirrors using the principle of total internal reflections. Still, it causes absorption and scattering, which could reduce the signal amplitude and intensity. According to [3], a cyclic interferometer is twice sensitive than Michelson's and the single- and multiple-reflections of the cyclic interferometer are compared as well.

As presented in [4-5], the number of reflections increase the number of fringes. By rotating the orientation of a half wave plate (HWP), the phase shift and clear fringes can be achieved. The refractive index measurement is proposed in this paper using multiple reflections generated by two equal-diameter mirrors with a PZT attached to one of them for signal control. The multiple interference fringes were achieved and used for the determinations of incident and reflected angles. A HWP was orientated at 0, π/2, π and 3π/2 rad respect to the reference plane to provide four interferograms that were later converted into intensities and phase retardations.

II. PRINCIPLE OF MULTIPLE REFLECTION OPERATIONS

Fig. 1 shows the schematic diagram of the proposed interferometric configuration. An input light with a wavelength of 632.8 nm is incident on the HWP oriented at an angle θ to the incident plane prior to travelling to the polarization beam splitter (PBS). The PBS then splits this beam into two orthogonal polarized beams, transmitted and reflected beams. The transmitted beam traveling along path 1 collides with mirror M_3 and is reflected several times between two mirrors, M_2 and M_3, having the same 25.4 mm diameter. The PZT attached to M_3 introduces a vibration driven by the signal from a function generator set at a frequency of 420Hz and amplitude of 5 V. The reflected beam along path 2 is first reflected by M_1 before undergoing into multiple reflections at M_2 and M_3. The two beams recombine at the PBS and pass through the quarter wave-plate (QWP) oriented at 45° to the reference, where the lights are transformed to right- and left-circularly polarized lights. The output signal out of the interferometer is captured by the CCD. As expressed in (1), the input electric field is generated from a monochromatic source through the HWP oriented at an angle θ to the incident plane before the beam is directed into the interferometer.

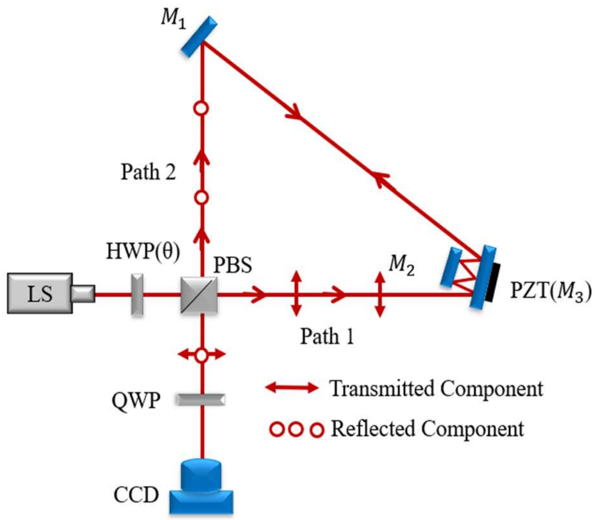

Fig.1. Schematic of the multiple reflections using modified Sagnac interferometer

$$E_i = HWP(\theta) = -i\begin{bmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{bmatrix} \tag{1}$$

The electric field vector of the transmitted light along path 1 can be represented by

$$E_{iT} = QWP \bullet PBS_T \bullet M_1 \bullet M_2 \bullet PZT(M_3) \bullet PBS_T \bullet E_i \tag{2}$$

$$QWP = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & -i \\ -i & 1 \end{bmatrix} \tag{3}$$

$$PBS_T = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \tag{4}$$

$$M = \begin{bmatrix} e^{i\delta_x} & 0 \\ 0 & e^{i\delta_y} \end{bmatrix} \tag{5}$$

$$E_{iT} = \frac{-i\cos 2\theta}{\sqrt{2}}\begin{bmatrix} e^{i(\delta_{3x}(t)+\delta_{2x}+\delta_{1x})-i\frac{\pi}{2}} \\ -ie^{i(\delta_{3x}(t)+\delta_{2x}+\delta_{1x})-i\frac{\pi}{2}} \end{bmatrix} \tag{6}$$

where M is the phase retardation introduced by the three mirrors M_1, M_2 and M_3, and δ represents the phase retardation in x- or y-direction. Whereas the electric field vector of the reflected light along path 2 can be determined with the same manner as follows.

$$E_{iR} = QWP \bullet PBS_R \bullet M_2 \bullet PZT(M_3) \bullet M_1 \bullet PBS_R \bullet E_i \tag{7}$$

$$E_{iR} = \frac{i\cos 2\theta}{\sqrt{2}}\begin{bmatrix} -ie^{i(\delta_{3y}(t)+\delta_{2y}+\delta_{1y})} \\ e^{i(\delta_{3y}(t)+\delta_{2y}+\delta_{1y})} \end{bmatrix} \tag{8}$$

Let $\alpha = \delta_{3x}(t)+\delta_{2x}+\delta_{1x}$ and $\beta = (\delta_{3y}(t)+\delta_{2y}+\delta_{1y})$, the electric field vector of the light output can be written as

$$E_T = \frac{1}{\sqrt{2}}\cos 2\theta\left\{\begin{bmatrix} 1 \\ -i \end{bmatrix}e^{i(\alpha-\frac{\pi}{2})} + \begin{bmatrix} 1 \\ i \end{bmatrix}e^{i\beta}\right\} \tag{9}$$

This E_T is the combinations of left-handed and right-handed circular polarizations. The phase shift is formed by rotating the HWP at the interferometer's input at the step of $\pi/2$. Therefore, the total electric field E_T at the output of interferometer is given by

$$E_T = \frac{-1}{\sqrt{2}}\left\{\begin{bmatrix} 1 \\ -i \end{bmatrix}e^{i(\alpha-\frac{\pi}{2})} + \begin{bmatrix} 1 \\ i \end{bmatrix}e^{i\beta}\right\} \tag{10}$$

III. INTERFERENCE RELATION

The basic intensity equation of the interference pattern between the two beams can be expressed as

$$I(x,y) = A_0(x,y) + A_1(x,y)\cos(\phi(x,y)+\delta) \tag{11}$$

where A_0, A_1 and ϕ are the average intensity, amplitude, and phase shift, respectively, and δ represents the phase step from the setting of the HWP. The interference intensities from four settings of HWP and the phase shift are presented in (12-15) and (16), respectively.

$$I_0(x,y) = A_O(x,y) + A_1(x,y)\cos\phi(x,y) \tag{12}$$

$$I_{90}(x,y) = A_O(x,y) - A_1(x,y)\sin\phi(x,y) + \pi/2 \tag{13}$$

$$I_{180}(x,y) = A_O(x,y) - A_1(x,y)\cos\phi(x,y) + \pi \tag{14}$$

$$I_{270}(x,y) = A_O(x,y) + A_1(x,y)\sin\phi(x,y) + 3\pi/2 \tag{15}$$

$$\phi(x,y) = \tan^{-1}\left(\frac{I_{270^\circ} - I_{90^\circ}}{I_{0^\circ} - I_{180^\circ}}\right) \tag{16}$$

IV. MULTIPLE REFLECTIONS AND REFRACTIVE INDEX MEASUREMENTS

Both transmitted and reflected beams traveling in the interferometer collided with the mirror M_3 at the air-glass interface. Some parts of the beams were partially reflected on the surface, while others were mainly reflected through the mirror, as depicted in Fig. 2. With the help of Snell's law, the angle of refraction α in the mirror is obtained by

$$\alpha = \sin^{-1}\left(\frac{\sin i}{n_{glass}}\right) \tag{17}$$

where, i is an incident angle and n_{glass} is a refractive index of glass (1.5). The refractive index of the mirror M_3 attached to PZT is achieved through the multiple reflections set up between the two reflecting mirrors M_2 and M_3 at a close distance. According to [6], the refractive index of a mirror n_m can be found by [6]

$$n_m = \left(\frac{t}{m\lambda}\right)\left[\frac{\alpha}{h}\right]^2 \tag{18}$$

where t, m, α, h, and λ are the mirror thickness, number of fringes, angle of refraction, a gap between M_2 and M_3, and light wavelength, respectively, as illustrated in Fig.2.

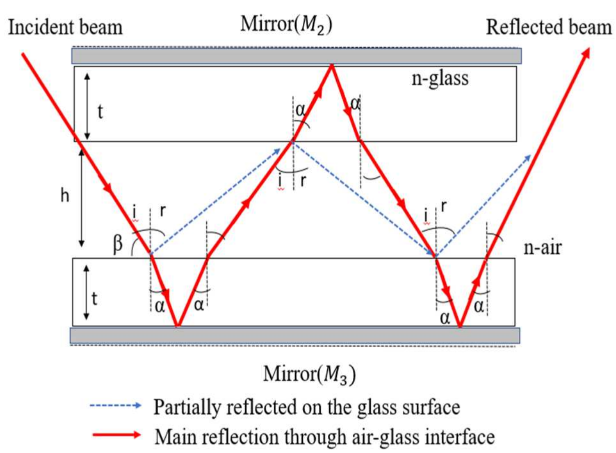

Fig. 2. Schematic of the multiple reflections between the two mirrors M_2 and M_3.

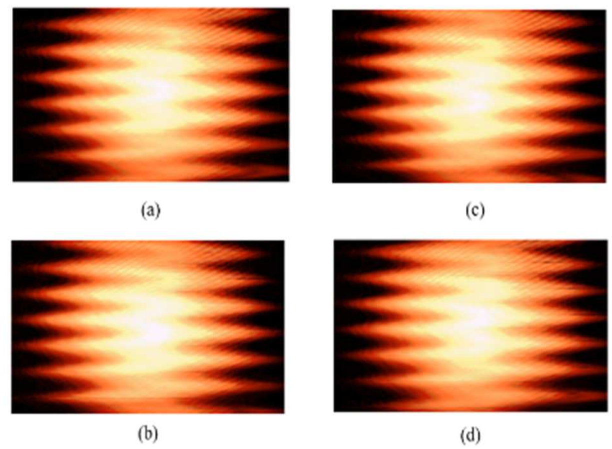

Fig. 3. An interferogram captured at the phase shift of (a) 0°, (b) 90°, (c) 180° and (d) 270°.

V. EXPERIMENTAL RESULTS AND DISCUSSIONS

A protected silver mirror from Thorlabs, Model:PF10-03-P01, with thickness and diameter of 6 mm and 25.4 mm, respectively, was used throughout the experiment. The space between two mirrors h is measured as 5 mm. A PZT is used to set a self-reference frequency at a specific vibration of 420 Hz. On the interference fringe pattern, the number of fringes m was counted to be 7 as shown in Fig. 3 The interferogram was detected by FHD 50MP 1080P USB HDMI industrial digital electronic video microscope camera and the image was then filtered in MATLAB to remove unwanted noises. Four identical interferograms were presented in Fig.3 as the HWP oriented at 0, $\pi/2$, π and $3\pi/2$ radians. The intensities of the respective interferograms were evaluated and summarized in Table 1. The value of phase shifts was calculated using (16) as 0.5 rad. The interferogram was further analyzed by converting the pixel decimal number (0-255) to an eight-bit binary code equivalence. The 2D phase maps for all feasible extractions were then obtained, as shown in Fig.4, starting with the least significant bit (LSB) and ending with the most significant bit (MSB) [7][8]. According to (17), the incident (i) and refracted angles (α) were 29° and 19.26°, respectively, With the help of (18), the refractive index of the protected silver mirror (n_m) was assessed to be 0.12511. The obtained value is close to that of silver of 0.13511 at λ = 633 nm [9], as the mirror is coated with silver.

TABLE 1. INTENSITIES AT VARIOUS PHASE SHIFT

Intensities (I)	Mean pixel	Min pixel	Max pixel
0	108.379	0	255
90	108.571	0	255
180	103.02	0	255
270	105.558	0	255

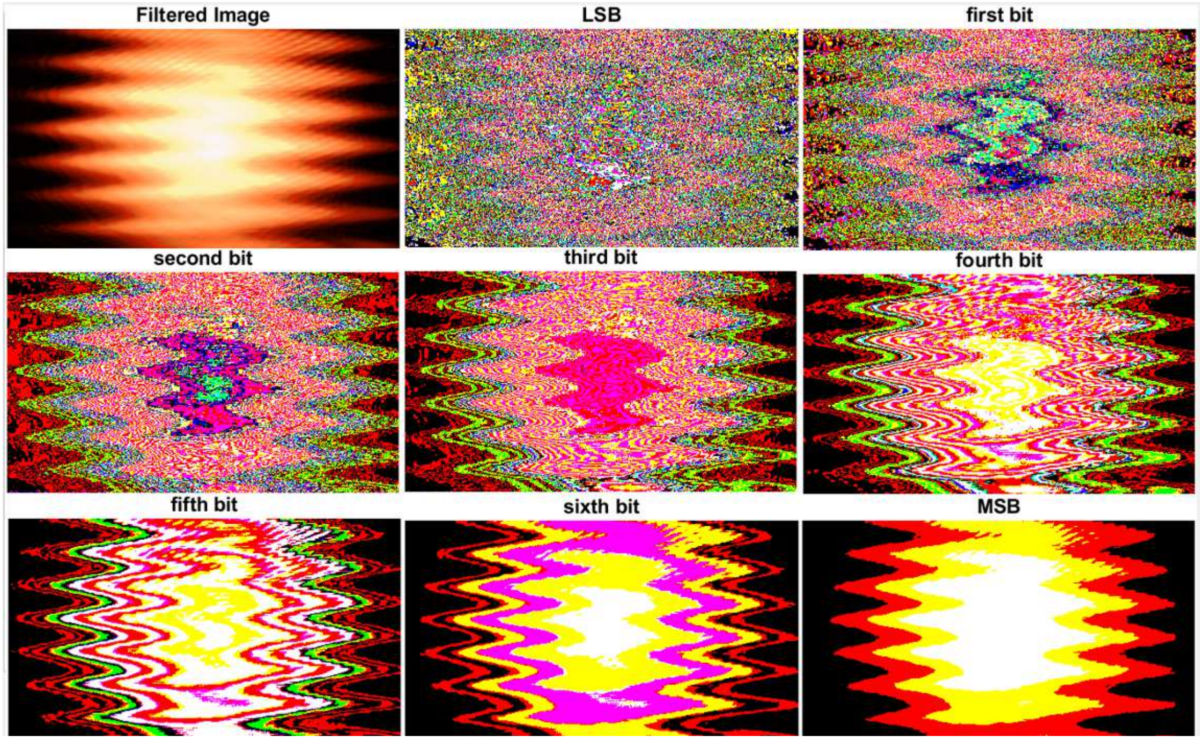

Fig. 4. The phase map extracted of the filtered image from the LSB to the MSB.

VII. CONCLUSION

The refractive index of a mirror attached to a PZT was achieved with the proposed scheme. The phase retardation was introduced by the mirror attached to PZT to produce the interference pattern as one of the fundamental parameters in the measurement. The proposed setup showed that the high sensitivity could be achieved with an automatic phase shifting, which produced multiple fringes at fixed tilt angle with a specific vibration frequency of 420Hz. The limitation of this setup was the fixed number of fringes, which was obtained and there was no effect on the interference output by inserting any transparent plate in the sensing arm. The refractive index of the coated silver mirror measured with our proposed modified Sagnac interferometer was found to be satisfactory when compared to the refractive index of the coated material.

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