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Thickness Measurement of Double Layer Transparent Material Using Triangular Path Cyclic Interferometer

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***Abstract*—The new scheme of a non-contact thickness measurement of multi-layer thin films with improving the sensitivity, accuracy, and performance of the measurement techniques is reported in this paper. Single- and double-layer film thickness measurement using different polarization states were studied. An alternative architecture using triangular cyclic interferometer with simultaneous measurements of reference and sample arms by splitting the beam at the output of interferometer is mathematically achieved. The polarization phase shift using two steps algorithm is observed, with steps shift of π/2 rad for a double layer sample. This shows that the polarization state has an impact on the thickness measurement of different thin-film materials.**

***Keywords*—thin-film thickness measurement, cyclic interferometer.**

I. Introduction

The performance of cyclic interferometer has been striving to improve in diverse architectures. A single layer thickness measurement of transparent was observed using rotating linearly polarized light. The outcomes of rotating linearly polarized lights without a sample is considered as the reference state to determine the phase shift of the polarized light and thickness measurement. [1] achieved the thickness of a single layer element inserted to alter light transmission is defined in terms of phase retardation. The method is complex to implement as it carried out an observation of reference and sample tests separately. Wave plate introduction in this experiments does not have any effect because rotating linearly polarized is considered as a reference stage. According to [2], the experiment addressed the issues encountered in the previous report. The model is modified with a non-polarizing beam splitter to split the incident beams to form reference and test beams. In this case, two beams are transmitted and reflected both in parallel and perpendicular to the incident beam. The two beams recombined at beam splitter after being reflected by the two mirrors and formed dual beams, each with perpendicular and parallel components of polarization.

In this paper, a new model is proposed for thickness determination of double layer, using triangular cyclic interferometer, to achieve simultaneous light intensity measurement of reference and sample arms. The principle of lateral shear is applied by using a single alignment to avoid any convergence or divergence of beams in the interferometric paths. The mathematic models of output light intensities at different settings of an analyzer are expressed in this report.

II. Principle Of Operation

The basic theory of the mathematical relationship of polarized light using the interferometric technique can be explained with the help of Jones calculus. The state of polarization is obtained from Jones vectors and Jones matrices of the optical elements. The electric field vectors E_x and E_y of propagating lights in two major directions, are given as functions of time (t) and distance (z) as

$$E_X = E_O \cos(wt - kz)\,\hat{x} \qquad (1)$$

$$E_y = E_O \cos(wt - kz)\,\hat{y} \qquad (2)$$

where E_0 is an amplitude of the signal, ω is an angular frequency, and k is a propagation constant. The proposed interferometric approach is depicted in Fig. 1. Light emitted by a laser travels through a half-wave plate (HWP) before being split into two beams by a polarization beam splitter (PBS). A HWP oriented at 22.5° relative to the plane of incidence, a reference plane, forms a linearly polarized light at 45° relative to the reference. The two beams out of a PBS are orthogonal (horizontal and vertical components) travelling along path 1 and 2. Two mirrors (M1 and M2) are placed to reflect the orthogonal beams as mirror 1 is attached to a piezoelectric transducer (PZT) driven by a periodic signal to automatically rotate a mirror, which changes a path length. The two beams combine at a PBS and are split again by a non-polarizing beam splitter or a cube beam splitter.

The quarter-wave plate (QWP) and an analyzer or a linear polarizer P (θ) are located in front of a charge-coupled device (CCD) camera in order to convert a phase-modulation into intensity modulation. A CCD camera, used in place of a photo detector, captures the fringes at the output of an interferometer. The phase retardation and thickness measurement using a double-layer transparent sample are analyzed by placing the analyzer at the output prior to the capture of output light fringes. The two-phase stepping algorithm is used to obtain light intensities out of an analyzer set at 0^{o} and 90^{o} to the reference plane. These intensities can be used to determine the phase retardation of the light created by the disturbance. The electric field of polarized light out of a HWP, E_h, can be expressed as

$$E_h=\frac{1}{\sqrt{2}}\begin{bmatrix}1\\1\end{bmatrix} \tag{3}$$

From Fig. 1, the light electric field vector E_{iT} travelling through a PBS with parallel polarization before reaching an analyzer with the route of (PBS→M2→PZT(M1)→PBS→PB→QWP) is given as [1]:

$$E_{iT} = \frac{1}{2}\begin{bmatrix}1\\-i\end{bmatrix} \exp(\delta_{m2}x(t) + \delta_{m1}x) \tag{4}$$

where, x is the light travelling distance, δ_{m1} and δ_{m2} are the phase retardation introduced by M1 and M2, respectively. Whereas, for the other part (PBS→PZT (M1)→M2 →PBS→PB→QWP), the electric field vector of reflected polarization (E_{iR}), travelling with distance y, can be given as.

$$E_{iR} = \frac{1}{2}\begin{bmatrix}-i\\1\end{bmatrix} \exp(\delta_{m1}y(t) + \delta_{m2}y) \tag{5}$$

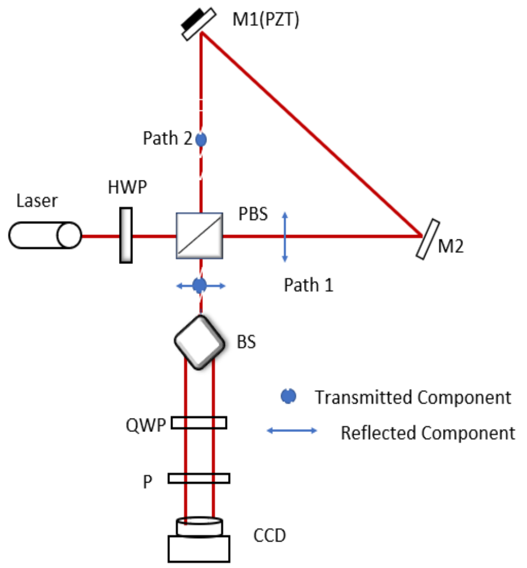

Fig.1. Proposed triangular path cyclic interferometer for phase shift measurement.

The total electric field E_{Tot} at the output prior to reaching an analyzer can be found by the summation of $E_{iT} + E_{iR}$ as

$$E_{Tot} = \frac{1}{2}\begin{bmatrix}1-i\\1-i\end{bmatrix} \exp[(\delta_{m2}x(t) + \delta_{m1}x) + (\delta_{m1}y(t) + \delta_{m2}y) \tag{6}$$

Equation (6) is the reference electric field and used to generate rotation of linearly polarized light as in [3].

III. Electric Field Vectors With Sample In Place

The previous section has mentioned about the electric field vector determination without any samples in place. This section demonstrates how the electric fields turn out as a sample is placed before a QWP in one arm as shown in Fig. 2.

The electric field vector (E_{iT_S}) of parallel polarization or transmitted route, when a sample is placed before a QWP, can be written as

$$E_{iT_S} = \frac{1}{\sqrt{2}}\begin{bmatrix}1 & -i\\-i & 1\end{bmatrix}\begin{bmatrix}T_1 & 0\\0 & T_2\, e^{i\Delta s}\end{bmatrix}\times\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\0 & 0\end{bmatrix} \begin{bmatrix}e^{i\delta_{m2}x(t)} & 0\\0 & e^{i\delta_{m2}y(t)}\end{bmatrix}\begin{bmatrix}e^{i\delta_{m1}x} & 0\\0 & e^{i\delta_{m1}y}\end{bmatrix} \begin{bmatrix}1 & 0\\0 & 0\end{bmatrix}\frac{1}{\sqrt{2}}\begin{bmatrix}1\\1\end{bmatrix} \tag{7}$$

where T_1 and T_2 are the transmission coefficient along the principal axes and Δ_s represents phase retardation introduced by the sample. The ultimate result of (7) can be expressed as

$$E_{iT_S}=\frac{1}{2}\begin{bmatrix}T_1\\-iT_1\end{bmatrix} e^{[\mathrm{i}(\delta_{m2}x(t)+\delta_{m1}x)]} \tag{8}$$

By working with the same manner as previous two equations but different direction, the electric field of perpendicular polarization (E_{iR_S}) can be found as

$$E_{iR_S}=\frac{1}{2}\begin{bmatrix}-iT_2\\T_2\end{bmatrix} e^{[\mathrm{i}(\delta_{m2}y(t)+\delta_{m1}y+\Delta s)]} \tag{9}$$

The total Electric Field (E_{TOT_S}) of light passing through a sample, being the summation of electric fields in Equation (8) and Equation (9), can be expressed as

$$E_{TOT_S}=\frac{1}{2}\begin{bmatrix}T_1\\-iT_1\end{bmatrix} e^{[\mathrm{i}(\delta_{m2}x(t)+\delta_{m1}x)]}+\frac{1}{2}\begin{bmatrix}-iT_2\\T_2\end{bmatrix}\times e^{[\mathrm{i}(\delta_{m2}y(t)+\delta_{m1}y+\Delta s)]} \tag{10}$$

By letting $\alpha = \delta_{m2}x(t) + \delta_{m1}x$ and $\beta = \delta_{m2}y(t) + \delta_{m1}y + \Delta s$, the output total electric field can be written as

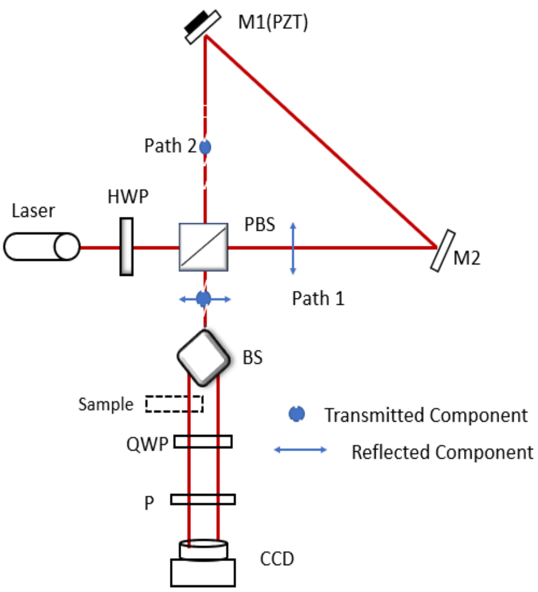

Fig.2. Triangular path cyclic interferometer with sample.

$$E_{TOT_S}=\begin{bmatrix} T_1\, e^{i\alpha} - iT_2 e^{i\beta} \\ -iT_1\, e^{i\alpha} + T_2 e^{i\beta} \end{bmatrix} \qquad (11)$$

IV. Light Intensities With Various Polarizer Settings

A polarizer placed at the output of interferometer is adjusted at 0°, 45°, 90°, and 135° to a reference. The total output of the field, when a linear polarizer is oriented at angles mentioned above, are given as in (13) – (16).

$$E_{out(0^o)} = \frac{1}{2}\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} T_1\, e^{i\alpha} - iT_2 e^{i\beta} \\ -iT_1\, e^{i\alpha} + T_2 e^{i\beta} \end{bmatrix} \quad (12)$$

$$E_{out(45^o)} = \frac{1}{2}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} T_1\, e^{i\alpha} - iT_2 e^{i\beta} \\ -iT_1\, e^{i\alpha} + T_2 e^{i\beta} \end{bmatrix} \quad (13)$$

$$E_{out(90^o)} = \frac{1}{2}\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} T_1\, e^{i\alpha} - iT_2 e^{i\beta} \\ -iT_1\, e^{i\alpha} + T_2 e^{i\beta} \end{bmatrix} \quad (14)$$

$$E_{out(135^o)} = \frac{1}{2}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} T_1\, e^{i\alpha} - iT_2 e^{i\beta} \\ -iT_1\, e^{i\alpha} + T_2 e^{i\beta} \end{bmatrix} \quad (15)$$

It is verified that (13) and (15) form complex matrices, which could not be simplified further to obtain the intensities at 45° and 135°. Whereas (12) and (14) yield scalar matrices for the intensities at 0° and 90°. The intensity is defined as $I(\theta) = E \times E^*$, where θ refers to the angle that a linear polarizer is set related to a reference plane. Output light intensity when a polarizer is adjusted at $\theta = 0°$ can be given as

$$\mathrm{I}\ (0^o) = \frac{1}{4}\begin{bmatrix} T_1 e^{i\alpha} & -iT_2 e^{i\beta} \\ & 0 \end{bmatrix}\begin{bmatrix} T_1 e^{i\alpha} & -iT_2 e^{i\beta} & 0 \end{bmatrix}$$

$$\mathrm{I}(0^o)= \frac{1}{4}\,[T_1^2 + T_1 T_2\, e^{-i(\beta-\alpha)} - iT_1 T_2\, e^{i(\beta-\alpha)} + T_2^2]$$

$$\mathrm{I}\ (0^o) = \frac{1}{4}[T_1^2 + T_2^2 + 2\, T_1 T_2\, sin\ (\beta - \alpha)] \qquad (16)$$

By substituting the values of α and β into equation (16), it yields

$$\mathrm{I}\ (0^o) = \frac{1}{4}\,[\,T_1^2 + T_2^2 +$$

$$2T_1T_2 \sin\,[(\delta_t(t) + \Delta_S)\ (\delta_t(t)] \qquad (17)$$

where $\square_t(\mathrm{t}) = \delta_{m2}x(t) + \delta_{m1}x = \delta_{m2}y(t) + \delta_{m1}y$ is a time varying phase called offset time. According to [1], this is the time varying phase of rotating linearly polarized light orientation, which depends on the rotation of the mirror using a PZT. A fixed polarizer at the output allows an automatic rotation of linearly polarized light that causes the initial phase of the apparatus to be offset. The time measurements of corresponding intensities are always relative to the offset time [2]. The time varying phase of (17) can be canceled out and is then expressed as.

$$\mathrm{I}\ (0^o) = \frac{1}{4}[T_1^2 + T_2^2\ + 2T_1T_2\ sin\ (\ \square\square)] \qquad (18)$$

By working with the same manner, light intensities with $\theta = 90°$ can be achieved as

$$\mathrm{I}\ (90^o) = \frac{1}{4}[T_1^2 + T_2^2 - \ 2T_1T_2\ sin\ (\ \square\square)] \qquad (19)$$

With the help of (18) and (19), the phase retardation of the proposed system can be found by

$$\square\square = sin^{-1}\left(\frac{\mathrm{I}(0^o) - \mathrm{I}(90^o)}{T_1T_2}\right) \qquad (20)$$

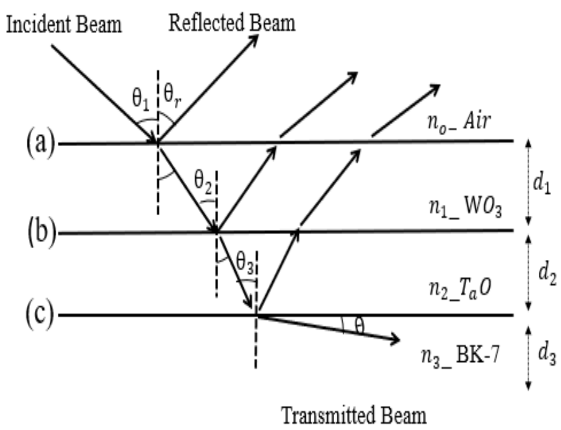

Fig. 3 Light transmission model through double-layer thin film [4].

V. Double Layer Transparent Sample

The proposed double-layer thin films is made up of BK-7 transparent substrate with refractive index n_3 of 1.5168 and thickness d_3. The thin layer of TaO is deposited on the BK-7 with thickness d_2 and refractive index n_2 of 2.136. Then, another film layer of WO_3 is grown on top of TaO layer with thickness d_1 and refractive index n_1 of 2.10 [4] as shown in Fig. 3, describing the transmission/reflection processes from lower to higher indices interfaces and vice versa. In Fig. 3, θ_1 and θ_r are the incident and reflected angles, respectively, whereas θ_2 and θ_3 are the refracted angles after the beams were transmitted from (a) to (c), accordingly. As phase retardation Δ proposed by [1] and intensities-phase relation in (20), the phase retardation of the double layer proposed in this experiment is defined as

$$\Delta = \frac{4\pi n d}{\lambda} \tag{21}$$

Following equation (21), the thicknesses of the two layers are given by.

$$d_1(WO_3) = \frac{\Delta_1 \lambda}{4\pi n_1} \tag{22}$$

$$d_2(T_aO) = \frac{\Delta_2 \lambda}{4\pi n_2} \tag{23}$$

where n_1 and n_2 are the refractive indices of the two layers deposited on the substrate, and Δ_1, Δ_2, d_1 and d_2 are the phase retardation and thickness values of the two layers, accordingly. The sample is placed after the interferometer, which makes the system applicable for microscopic measurements as reported in [5].

VI. Discussion

The mathematic analysis of essential electric fields for phase retardation determination are expressed and explained in previous sections. The two-step algorithm of acquiring two light intensities from 0^o and 90^o setting of an analyzer is crucial as those intensities are later used to obtain the phase retardation, which, eventually, is converted to the film thickness. The proposed mode is mathematically proved to be effective for thin-film thickness measurement. The experiment will be conducted in the next phase of our work. The main contribution of this system is simultaneous measurement of the two arms interferometer through an automatic phase shift difference with the help of a PZT.

VII. Conclusion

Alternative designs of triangular and path cyclic interferometer are proposed using two-step algorithm for phase shift measurements by placing double layers thin films of WO_3/TaO at the output of the interferometer. The step-by-step of a determination of phase retardation introduced by the sample with double layer thin films is described and achieved in this paper. The setup has a good potential to be applicable in deformation measurements of microscopic sample for several applications including disease detections.

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