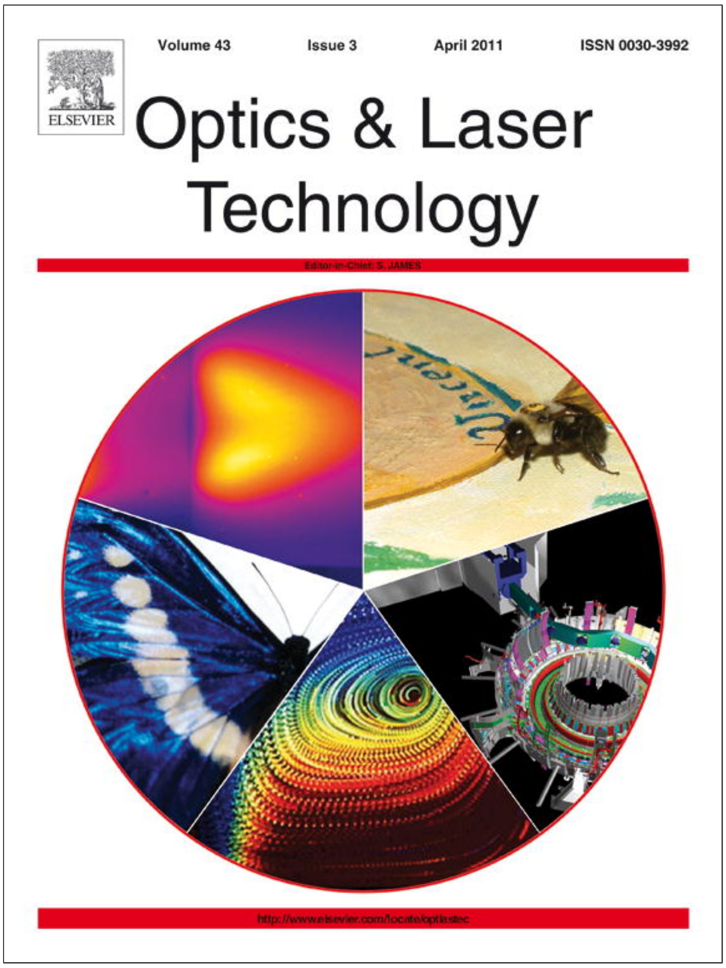
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The rotating linearly polarized light from a polarizing Mach–Zehnder interferometer: Production and applications

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ABSTRACT

The production and control of a required polarization state are very important in optical measurements such as thin film surface characterization, ensuring continuity of data measurement and speedy acquisition. In this paper, a polarizing Mach–Zehnder interferometer is used to generate rotating linearly polarized light. Theoretical analysis and experimentation on the proposed configuration show good agreement. Applications of the rotating linearly polarized light generated from the proposed polarizing interferometer are demonstrated. The applications include measurements of the phase retardation of several wave retarders. The results from the preliminary investigation show a promising performance of the proposed setup.

1. Introduction

Polarization is an important property of light and can be employed as a probing tool to characterize both bulk samples and thin film surfaces. By using linearly polarized light, simplified mathematical treatments such as Jones matrix algebra can easily be used to verify the experimental outputs. The well known ellipsometry technique used to characterize materials is based on the study of the transformation of the polarization states of light incident and reflected (or transmitted) from a sample [1]. By comparison of the incident and reflected (or transmitted) polarization states of light beams in terms of amplitude and phase, an appropriate theory, such as the theory of Fresnel reflection, can be used to calculate the corresponding optical properties of the sample under investigation [2]. The measurement process may be divided into two parts, consisting of the production of a known and controllable state of linearly polarized light and the analysis of the polarized light after the light–material interaction. Rotating linearly polarized light may be utilized in various optical measurements such as birefringence, optical activity, thin film measurement [3], current [4] and in a wide range of light scattering experiments [5]. A number of configurations for the production of rotating linearly polarized light have been reported. Early configurations used a rotating polarizer [6] or a rotating half-wave retarder [7]. The generation of the linearly polarized light can also be achieved by employing one or two acoustooptic cells [3,8]. With this particular arrangement, two modulated circularly polarized beams that differ in frequency and handedness are created. The circularly polarized beams with opposite senses then superimpose to give the rotating linearly polarized light. The technique suggests that any two-arm interferometer will serve the same purpose. Michelson and Mach–Zehnder interferometers can be appropriately arranged to function as sources of the rotating linear polarized light. The use of a Michelson interferometer as a global polarization state generator was proposed, producing polarized light in known states of polarization, including rotating linearly polarized light [9]. Several researchers have also modified and developed Mach–Zehnder interferometers to generate known states of polarized light, including rotating linear polarization. An unbalanced Mach–Zehnder interferometer using a time-dependent phase delay between two temporally coherent orthogonal circularly polarized states to produce continuously rotating linearly polarized light was proposed and successfully tested [10]. An alternative configuration of the modified Mach–Zehnder interferometer using an electro-optic modulator was proposed [11]. The configuration can provide two circularly polarized beams with opposite senses that may potentially be arranged to produce rotating linearly polarized light. The Mach–Zehnder interferometer based technique is very interesting as it is a simple arrangement and offers simple signal processing. Therefore, in this research, the linear polarization state generator based on the so-called polarizing Mach–Zehnder interferometer is proposed. Although the proposed arrangement is based on the same principle as the unbalanced Mach–Zehnder interferometer, some modifications are made to further simplify the arrangement, for example using a one-piece optical device that includes several relevant polarizing

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components instead of discrete optical components. Apart from arranging the polarizing Mach–Zehnder interferometer to generate rotating linearly polarized light and to control the rotation speed and orientation, the output from the interferometer is also employed to determine the phase retardations of wave retarders. Although some studies [12,13] have already been shown that the Mach–Zehnder interferometer yields reliable and interesting results in phase retardation measurement, there is still room for improvement especially in terms of the optical setup and signal processing. In terms of the setup, the proposed polarizing Mach–Zehnder interferometer employs polarizing beam splitters as main optical components. The polarizing component usage ensures that a high degree of polarization can be achieved. Also, this arrangement should allow one to gain insight into how the interferometer works and to conveniently diagnose problems such as the reduction of the degree of polarization, which may occur. In terms of the signal processing, the two outputs from the proposed interferometer can be used to provided reference and sample signals, in which the former can be used to track the latter by way of a simple phase comparison. This allows the direction of the azimuth of the rotating linearly polarized light beam after an interaction to be identified. The verification of the state of polarized light produced with this arrangement is conducted in terms of the Stokes parameters. The measured values from the output are checked against theoretical predictions derived from the Jones matrix algebra. Known samples, namely quartz wave retarders and liquid crystal variable retarder, that introduce fixed and variable phase shifts are investigated by the setup.

2. Theoretical background

2.1. The generation of the rotating linearly polarized light

As can be seen from Fig. 1, a properly adjusted half-wave plate, HW, oriented at 45° with respect to the transmission axis of the PB1, is introduced after the HeNe laser source, LS. The Jones vector of the incident linear polarized light on the Mach–Zehnder interferometer is given as

$$E_0 = \frac{E}{\sqrt{2}}\begin{bmatrix}1\\1\end{bmatrix}\exp[i(\phi_0-\omega t)] \tag{1}$$

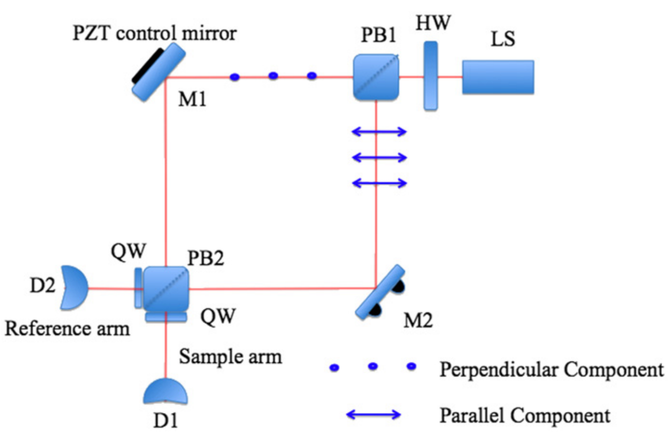

Fig. 1. This illustrates the experimental arrangement of the proposed polarizing Mach–Zehnder interferometer. LS = HeNe laser source, HW = half wave retarder, PB1 = polarizing beamsplitter, PB2 = polarizing beamsplitter with quarter wave retarders (QW) attached at the output sides, M1,2 = mirrors, PZT = piezoelectric transducer, D1,2 = photodetectors.

where E is the amplitude of the input light. ϕ_0 is the initial phase, and ω is the angular frequency of the input light.

The polarizing beamsplitter PB1 separates the input light into two orthogonally linear components. The Jones matrix of PB1 may be described with two transformation matrix references. They are, respectively,

$$T_{PB1T} = \begin{bmatrix}1 & 0\\0 & 0\end{bmatrix} \tag{2a}$$

and

$$T_{PB1R} = \begin{bmatrix}0 & 0\\0 & 1\end{bmatrix} \tag{2b}$$

where subscripts T and R refer to transmission and reflection components, respectively.

This polarization division is due to the physical properties of PB1. Because of the high extinction ratio of PB1, in practice, nearly perfect linear polarized light from the transmission and reflection can be achieved.

Therefore, the transmitted light E_{1T} from PB1 is written as

$$E_{1T} = T_{PB1T}E_0 = \frac{E}{\sqrt{2}}\begin{bmatrix}1\\0\end{bmatrix}\exp[i(\phi_0-\omega t)] \tag{3a}$$

Likewise, the reflected light E_{1R} from PB1 is given as

$$E_{1R} = T_{PB1R}E_0 = \frac{E}{\sqrt{2}}\begin{bmatrix}0\\1\end{bmatrix}\exp[i(\phi_0-\omega t)] \tag{3b}$$

It is evident that E_{1T} and E_{1R} are linearly parallel to the corresponding transmission axes of PB1. Each light beam is reflected by a mirror once (M1 or M2 depending on its path) before being incident on PB2. Both beams experience different additional phases corresponding to their propagating paths from PB1 to PB2.

E_{1T} experiences a phase shift of ϕ_1 from propagation along the path PB1 → M1 → PB2. This beam is modulated sinusoidally by the modulating mirror M1 that is attached to a modulator. The Jones vector of the beam reflected from M1 is written as

$$E_{1T'} = \frac{E}{\sqrt{2}}\begin{bmatrix}1\\0\end{bmatrix}\exp[i((\phi_0+\phi_1)-(\omega+\Omega)t)] \tag{4a}$$

It should be noted that the modulator imposes a frequency shift Ω onto the reflected beam.

Similarly, the beam reflected from M2 can be represented by $E_{1R'}$ and it experiences a phase shift ϕ_2 introduced by the propagation path PB1 → M2 → PB2

$$E_{1R'} = \frac{E}{\sqrt{2}}\begin{bmatrix}0\\1\end{bmatrix}\exp[i((\phi_0+\phi_2)-\omega t)] \tag{4b}$$

Therefore, PB2 allows the electric field $E_{1T'}$ to be transmitted and $E_{1RS'}$ to be reflected to the output of the interferometer where photodetector D1 is placed. It should be noted that PB2 is composed of a polarizing beam splitter and two pieces of quarter wave retarders oriented at 45° with respect to the transmission axis of the beam splitter. This optical device is, in fact, the Agilent 10705A single beam interferometer. With the properly attached wave retarders, the alignment of the optical components becomes more convenient.

The Jones vector of one output from the PB2 can be described as

$$E_2 = QE_{1T'} + QE_{1R'} \tag{5}$$

where Q is the Jones matrix of the attached quarter wave retarder with its fast axis oriented at 45° with respect to the transmission axis of the beam splitter.

The matrix Q is given by

$$Q = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & i \\ i & 1 \end{bmatrix} \tag{6}$$

Therefore, the Jones vector E_2 is found to be

$$E_2 = \frac{E}{2}\begin{bmatrix} 1 \\ i \end{bmatrix}\exp\left[i((\phi_0+\phi_1)-(\omega+\Omega)t)\right] + \frac{E}{2}\begin{bmatrix} i \\ 1 \end{bmatrix}\exp\left[i((\phi_0+\phi_2)-\omega t)\right] \tag{7}$$

The output E_2 represents a combination of two orthogonally circular polarized beams which then combine to form rotating linearly polarized light.

The output beam in Eq. (7) can be described by

$$E_2 = \frac{E}{2}\begin{bmatrix} \exp[i(\Delta\phi-\Omega t)]+i \\ \exp\left[i\left(\Delta\phi-\Omega t+\frac{\pi}{2}\right)+1\right] \end{bmatrix}\exp\left[i(\phi_0+\phi_2-\omega t)\right]$$
$$= E\begin{bmatrix} \sin\frac{1}{2}\left(\Delta\phi-\Omega t+\frac{\pi}{2}\right) \\ \cos\frac{1}{2}\left(\Delta\phi-\Omega t+\frac{\pi}{2}\right) \end{bmatrix}\exp\frac{i}{2}\left(2\phi_0+\phi_1+\phi_2-(\Omega+2\omega)+\frac{\pi}{2}\right) \tag{8}$$

where $\Delta\phi = \phi_1-\phi_2$ is the initial phase difference between two orthogonally linear polarized beams. Generally, the expression of the output as shown in Eq. (8) is the rotating linearly polarized beam whose speed depends on the modulation frequency Ω.

It is interesting to show, as follows, that the intensity I of the output light detected directly by the photodetector is stable

$$I = E_2E_2^*$$
$$= E^2\left[\sin^2\left(\Delta\phi-\Omega t+\frac{\pi}{2}\right)+\cos^2\left(\Delta\phi-\Omega t+\frac{\pi}{2}\right)\right] = E^2 \text{ or constant} \tag{9}$$

where E_2^* is the complex conjugate of E_2.

This then provides a simple way to check the expected state of polarization of the output beam. In practice, PB2 allows not only the major beam or high-intensity rotating linearly polarized light at the output side D1, but also the minor beam or low-intensity one with exactly the same state of polarization to be detected by D2. This is due to the imperfection of the polarizing beam splitter. In some optical experiments, this minor beam is probably unwanted, but in this arrangement this beam offers a way to simplify the signal processing. This is because the low-intensity portion can be used as a reference signal which acts effectively as a temporal pointer to identify states of polarization of the light detected by D1.

2.2. The application of the rotating linearly polarized light

To use the rotating linearly polarized light as a probing tool to measure the optical properties, such as phase retardation, of a sample, a simple signal processing approach is introduced.

To simplify the mathematical analysis, Eq. (8) representing the output beam from the interferometer may be rewritten as

$$E_2^{\phi'} = \begin{bmatrix} \sin\phi' \\ \cos\phi' \end{bmatrix}\exp(i(\phi'+\varepsilon)) \tag{10}$$

where $\phi' = 1/2(\Delta\phi-\Omega t+\pi/2)$ and this angle corresponds to the azimuth of the output beam with respect to a reference and $\varepsilon = \phi_0+\phi_2-\omega t$.

The preliminary application is chosen to be the transmission mode measurement of known birefringent samples. The sample, such as a wave retarder or liquid crystal variable retarder, is placed between PB2 and D1 in the so called sample arm of the polarizing Mach–Zehnder interferometer. The Jones matrix of the sample S is given as

$$S = \begin{bmatrix} T_1 & 0 \\ 0 & T_2\exp(i\delta) \end{bmatrix} \tag{11}$$

where T_1 and T_2 are the transmission coefficients in the principal axes of the sample and δ is the phase retardation introduced by the sample. The Jones vector of the transmitted light beam E_3 after the sample can be calculated from

$$E_3^{\phi'} = SE_2^{\phi'} = \begin{bmatrix} T_1\sin\phi' \\ T_2\cos\phi'\exp(i\delta) \end{bmatrix}\exp(i(\phi'+\varepsilon)) \tag{12}$$

In order to recover the phase retardation of the sample, polarizer (P1) is introduced next to the sample and in front of the photodetector D1 (Fig. 2).

Generally, the Jones matrix of a polarizer with its transmission axis oriented at an angle θ with respect to a reference direction is given as

$$P_{1,\theta} = \begin{bmatrix} \cos^2\theta & \cos\theta\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta \end{bmatrix} \tag{13}$$

The Jones vector of the transmitted light after the $P_{1,\theta}$ can be written as

$$E_{4,\theta}^{\phi'} = P_{1,\theta}E_3^{\phi'} = \begin{bmatrix} T_1\sin\phi'\cos^2\theta+T_2\cos\phi'\cos\theta\sin\theta\exp(i\delta) \\ T_1\sin\phi'\sin\theta\cos\theta+T_2\cos\phi'\sin^2\theta\exp(i\delta) \end{bmatrix}\exp(i(\phi'+\varepsilon)) \tag{14}$$

By choosing the orientation of the polarizer $P_{1,\theta}$ to be at 45° with respect to a reference direction, $E_{4,\theta}^{\phi'}$ may be simplified to

$$E_{4,45^0}^{\phi'} = \begin{bmatrix} \frac{T_1}{2}\sin\phi'+\frac{T_2}{2}\cos\phi'\exp(i\delta) \\ \frac{T_1}{2}\sin\phi'+\frac{T_2}{2}\cos\phi'\exp(i\delta) \end{bmatrix}\exp(i(\phi'+\varepsilon)) \tag{15}$$

Eq. (15) represents the output beam which is incident on the photodetector D1. The intensity obtained from the photodetector

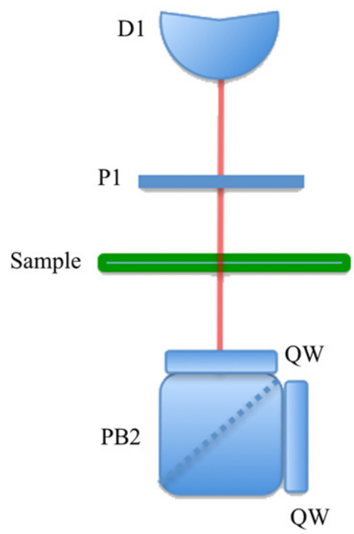

Fig. 2. A polarizer P1 is introduced into the sample arm of the polarizing Mach–Zehnder interferometer between the sample and photodetector D1.

becomes

$$I^{\phi'}_{4,45^\circ} = E^{\phi'}_{4,45^\circ} E^{\phi'*}_{4,45^\circ} = \frac{1}{2}\left[T_1^2 \sin^2\phi' + T_2^2 \cos\phi' + T_1 T_2 \sin 2\phi' \cos\delta\right] \quad (16)$$

where $E^{\phi'*}_{4,45^\circ}$ is the complex conjugate of $E^{\phi'}_{4,45^\circ}$.

The azimuth ϕ' of the output beam from the interferometer can simply be controlled by the modulation. Three specific values of azimuths are chosen. They include $\phi'=0^\circ$, 45° and 90°.

With a value of $\phi'=45^\circ$, the intensity $I^{\phi'}_{4,45^\circ}$ can be written as

$$I^{45^\circ}_{4,45^\circ} = \frac{1}{2}\left[\frac{T_1^2}{2} + \frac{T_2^2}{2} + T_1 T_2 \cos\delta\right] \quad (17)$$

When the azimuth of the rotating linearly polarized beam is set at $\phi'=0^\circ$ and 90°, the corresponding intensities become

$$I^{0^\circ}_{4,45^\circ} = \frac{T_2^2}{2} \quad (18)$$

and

$$I^{90^\circ}_{4,45^\circ} = \frac{T_1^2}{2} \quad (19)$$

By substitution T_1^2 and T_2^2 from Eqs. (18) and (19) into Eq. (17), the intensity becomes

$$I^{45^\circ}_{4,45^\circ} = \frac{I^{0^\circ}_{4,45^\circ}}{2} + \frac{I^{90^\circ}_{4,45^\circ}}{2} + \sqrt{I^{0^\circ}_{4,45^\circ} I^{90^\circ}_{4,45^\circ}} \cos\delta \quad (20)$$

Because $I^{0^\circ}_{4,45^\circ}, I^{45^\circ}_{4,45^\circ}$ and $I^{90^\circ}_{4,45^\circ}$ can be determined experimentally, the phase retardation δ of the sample can subsequently be worked out. For the reference arm, the P2 is placed between the PB2 and D2. A similar signal pattern with lower intensity can be observed.

At this point, a description of the reference signal is needed. As stated earlier, there are two rotating linearly polarized beams output from the polarizing Mach–Zehnder interferometer. One beam from the sample arm can be used as a probing tool as already described, while the other from the reference arm is suitable for use as a reference. The main function of the reference is to identify the state of polarization of the sample beam by using the common time base of both output beams as a pointer. Fig. 3(a) shows that both the sample and reference signals are in phase. The peaks and troughs of the reference signal originate from the rotation of the linearly polarized beam being parallel and perpendicular to the transmission axis of the reference polarizer P2. This should be noted that the direction of the P2 transmission axis is considered as the reference direction for the signal processing. The sample signal is also generated in the same manner. The Fig. 3(a) indicates that, at this moment, P1 and P2 are parallel to each other. If the reference signal remains undisturbed, any change that occurs in the sample signal can be easily identified by referring back to the peaks and troughs of the reference signal. In Fig. 3(b), the P1 in the sample arm is rotated by 45° with respect to the P2. This is evident that the sample signal is shifted relative to the reference signal. By determining the common time corresponding to the peaks and troughs in the reference, the intensities of the sample signal that correspond to the parallel and perpendicular directions; i.e., 0° and 90° with respect to the reference direction can be straightforwardly decided. For instance, in Fig. 3(b), because the incident light on the P1 without a sample is the rotating linearly polarized light with a constant amplitude. Thus, the amplitude of incident light beams at 45° and −45° to the direction of the P1 transmission axis (corresponding to perpendicular and parallel directions, respectively, relative to the reference direction) are found to be exactly the same.

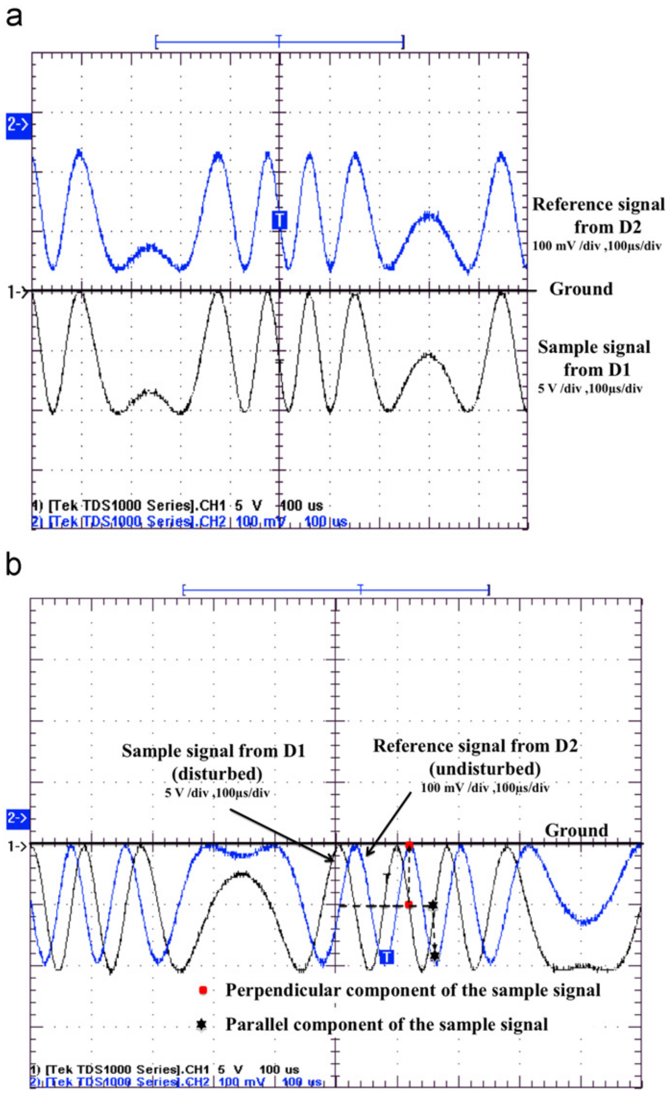

Fig. 3. (a) Sample signal and reference signals are in phase and (b) Sample signal and reference signals are out of phase. In both graphs, the horizontal axis corresponds to the time base and the vertical axis corresponds to the signal intensity.

3. Experimental setup

3.1. Interferometer configuration

The well-known Mach–Zehnder interferometer is shown in Fig. 1. A HeNe laser with $\lambda=632.8$ nm is used as a light source. The output of the laser is linearly polarized. With the help of a HW, the orientation of the polarized light can be rotated to any desired orientation. Because PB1 is polarizing, the most suitable orientation of the input light is at 45° with respect to the transmission axis of the beam splitter. The incident light is then equally divided into two orthogonal linear polarized light beams. Each polarized light beam is reflected once by M1 or M2 and then they combine at PB2. The linear polarized beam along the path PB1→M1→PB2 is parallel to one transmission axis of PB1; while, the linear polarized beam along the path PB1→M2→PB2 is parallel to the other transmission axis of PB1. PB2, Agilent 10705 A with a QW attached oriented at 45° with respect to the PB2 axes at each output side, transforms the two orthogonally linear input beams into two circularly polarized light beams.

The circularly polarized beams then combine to give a linearly polarized beam. With an inherent phase difference among the combining beams, the azimuth of the output linear polarized beam is set at an angle with respect to a reference direction. In this arrangement, the azimuth of the linear polarized light can be adjusted appropriately by modulating the mirror M1 with piezo-electric transducer (PZT). Because the modulation introduces a continuous phase difference between two circular polarized light beams, this results in the rotation of the linearly polarized output beam. It should be noted that each output of PB2 provides exactly the same polarization state. Only intensities of the two beams are different. The beam with higher intensity, the so called the sample beam, is detected by D1, whereas, the other beam, the so called the reference beam, is detected by D2. Their states of polarization are proved to be rotating linearly polarized by an output observation suggested by Eq. (9) and the Stokes parameters analysis carried out after the arrangement. By introducing two polarizers P1 and P2 at outputs of the polarizing interferometer, similar signals which are the sample and reference signals can be observed. This should be noted that by adjusting appropriate amplitude and frequency of the PZT modulating signal, the linear polarized beam can be achieved. In this experiment, the amplitude and frequency of the modulating sinusoidal signal were chosen to be 9.37 V and 900 Hz, respectively.

3.2. Prearrangement and signal determination

Because Eq. (20) is the key expression to determine the phase retardation of a sample under investigation, appropriate means are designed to measure intensities $I^{0^\circ}_{4,45^\circ}$, $I^{45^\circ}_{4,45^\circ}$ and $I^{90^\circ}_{4,45^\circ}$. The steps of the prearrangement before determining the required intensities are described as follows:

(1) Before a sample is introduced, the polarizers P1 and P2 in both arms are adjusted to achieve in-phase signals from D1 and D2. The times corresponding to the peaks and troughs of the reference signal provide a tool for recovering appropriate polarization state of the signal in the sample arm. The signals can be observed and recorded on-line for a further signal processing by a Tektronix oscilloscope model TDS 220.
(2) A sample such as a known wave retarder is inserted between the PB2 and the P1. By doing so, the signal from the sample arm is normally changed in terms of amplitude and phase due to the properties of the inserted sample properties.
(3) To set the optic axis of the sample to be parallel to the transmission axis of P1 (or P2), appropriate adjustment of the sample angular position is made. This would allow the orientation of the sample to be known.
(4) To obtain the transmitted signal with the phase information introduced by the sample, the incident linear polarized beam has to impinge on the sample when the orientation of the beam is at 45° with respect to the optic axis of the sample. Because the incident linearly polarized light on the sample is rotating, each revolution of the light provides all three required intensities; namely, $I^{0^\circ}_{4,45^\circ}$,$I^{45^\circ}_{4,45^\circ}$ and $I^{90^\circ}_{4,45^\circ}$, as seen from Fig. 4. A proper angular position of the P1 at 45° with respect to its initial position is set so as to obtain the required data.
(5) Now, the so called sample signal is clearly shifted relative to the reference signal.
(6) To determine the intensity corresponding to orientation 0°, 45° and 90° with respect to the optic axis of the sample, the reference signal is used. Since the optic axis of the sample is kept to be parallel to the transmission axis of the P2. Therefore, the peaks and troughs of the reference signal correspond to the sample intensities at 0° and 90° relative to the reference.

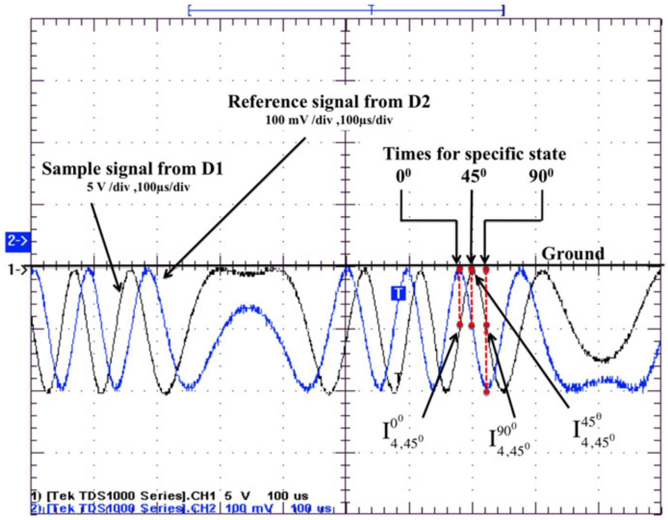

Fig. 4. Reference and sample signals are displayed on the oscilloscope and also states of the sample output beam can be determined by the reference signal.

4. Experimental results

The rotating linearly polarized light generated by the polarizing Mach–Zehnder interferometer can be used to measure phase shift introduced by a wave retarder. A set of known wave plates were used as samples to evaluate the application performance of the polarizing interferometer. The experiment starts with the production and study of the characteristics of the output in terms of the degree of linear polarization (DOP) and ellipticity (e). This is followed by the measurement of the phase shift introduced by known retarders.

4.1. Characteristics of the output

The expected output from the polarizing interferometer is rotating linearly polarized light. Theoretically, the DOP and e of the output are 100% and 0.0, respectively. These two values are set as preliminary targets to achieve. However, in practice, several unavoidable factors such as the imperfection of the optical devices and limited precision of the angular adjustments of optical devices prevent the expected achievements.

The characteristics of the output beams were determined in terms of four Stokes parameters. The Stokes parameters were measured using the LCPM-3000 liquid crystal polarimeter from Meadowlark Optics. The Stokes parameters were found to be $S_0=9.46\pm0.15$ V, $S_1=9.42\pm0.14$ V, $S_2=-0.006\pm0.001$ V and $S_3=0.058\pm0.002$ V. By substitution these parameters into the following equations, the DOP and e [2] can be determined

$$\mathrm{DOP}=\frac{\sqrt{S_1^2+S_2^2+S_3^2}}{S_0}\times 100\% \tag{21}$$

$$e=\tan\left(\frac{1}{2}\left\{\sin^{-1}\left[\frac{S_3}{\sqrt{S_1^2+S_2^2+S_3^2}}\right]\right\}\right) \tag{22}$$

The DOP and e of the output beams are measured to be $(99.6\pm0.3)\%$ and 0.0030 ± 0.0002, respectively.

This is obvious that the state of the output can reasonably be considered as the linearly polarized beam.

4.2. Phase shift determination of fixed retarders

In this part, two known retarders are examined so as to check the performance of the arrangement. In brief, the retarders under investigation are composed of two fixed phase-shift devices; i.e., quartz quarter and half wave retarders. The sample is introduced into the sample arm by properly placing it between the PB2 and P1. The optic axis of the sample is carefully adjusted to a proper orientation which is parallel to the system reference defined by the direction of the P2 transmission axis. The setup for determining the sample phase shift is illustrated in Fig. 5.

To evaluate the phase shift of the sample, the well known ellipsometric parameters ψ and Δ are measured. This should be noted that the measurements of the parameters are conducted in the transmission manner. Theoretically, ψ and Δ are described the change in amplitude and phase in the transmitted light beam. Clearly, the phase shift introduced by the retarder is equivalent to Δ. In addition, the value of ψ also provides information on the orientation of the incident light beam. By measuring the intensities as described in Eq. (20), the phase shift can be calculated. To evaluate ψ which is defined to be $\tan^{-1}\left(I^{0^\circ}_{4,45^\circ}/I^{90^\circ}_{4,45^\circ}\right)$, intensities as indicated by Eqs. (18) and (19) have to be measured. Table 1 shows measured values of cos Δ and ψ of the quarter and half wave retarders.

This should be noted that the values of the measured phase shift are reported in terms of cos Δ instead of just Δ. This is because a small fluctuation of the measured values around true values, especially in the case of the half wave retarder, gives uncalculable values of Δ. However, the variation is closely distributed around a mean value of cos Δ which is very close to the theoretical one for both samples. To ensure that the proposed arrangement potentially be applied to study specific sample characteristics, a further test on the measurement of a sample phase shift is needed. The details of the experiment will be discussed in Section 4.3. Because of the definition of the ψ parameter, the value of ψ in this case is expected to be 45° which means that the two orthogonal incident beams possess the same magnitudes. This should also imply that the incident beam is actually rotating linearly polarized. Consequently, the experimental results clearly confirm that the polarization state of the incident light on the retarders is in fact rotating linear.

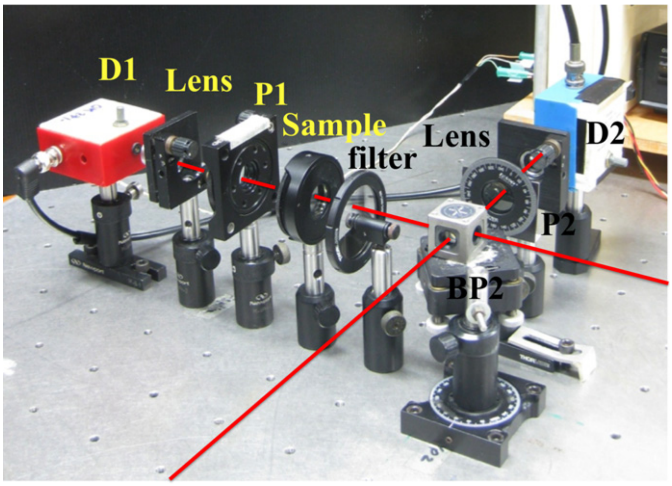

Fig. 5. The arrangement for the phase retardation measurement in the sample arm of the polarizing Mach–Zehnder interferometer. The sample is inserted between the PB2 and P1. (Please refers to Fig. 1 for the abbreviation meanings).

Table 1
Theoretical and measured values of ψ and cos Δ of known quartz retarders.

Retarders (at 632.8 nm)	Theoretical values		Measured values	
	cos Δ	ψ (deg.)	cos Δ	ψ (deg.)
Quarter wave	0.000	45.0	0.003 ± 0.001	44.7 ± 0.1
Half wave	−1.000	45.0	-1.016 ± 0.018	45.4 ± 0.6

4.3. Phase shift determination of the liquid crystal variable retarder

Liquid crystals become a good choice for variable retarders. They allow a continuous and convenient control of the phase shift by way of applying appropriate voltages. The benefit of using such a variable retarder like the liquid crystal retarder is that the performance of the proposed interferometric arrangement can be thoroughly assessed in a wider range of phase shift determination. Fig. 6 shows a typical graph of the relationship between the applied voltages and corresponding ellipsometric parameters ψ and Δ. The results shown in this graph were obtained from introducing the liquid crystal variable retarder to the sample arm of the proposed polarizing interferometer. The liquid crystal used in this research is the nematic liquid crystal variable retarder from Meadowlark optics.

To make sure that the measured values obtained in this experiment are correct and potentially useful, two specific conditions of the liquid crystal variable retarder performing as quarter and half wave retarders are closely examined.

According to the experimental results, the applied voltages of 1.77 and 2.34 V were found to make the liquid crystal variable retarder to behave as half and quarter wave retarders, respectively. Thus, a separate sample experiment was conducted. By simply shining appropriately oriented linear light beam onto the liquid crystal variable retarder with a specifically applied voltage, circular polarized light beam or linear polarized light beam with a new azimuth can be observed. These expected results were clearly proved by monitoring the output light at such specific applied voltages by a polarimeter. Fig. 7(a) clearly shows that at the applied voltage of 2.34 V, the liquid crystal transforms the linearly polarized light into circularly polarized light. In addition, at the applied voltage of 1.77 V, the polarization state of the output light remains linear but its orientation is changed from the input orientation by 45° (Fig. 7(b)).

5. Conclusions

The polarizing Mach–Zehnder interferometer was proposed to generate the rotating linearly polarized light. The arrangement is based on a conventional Mach–Zehnder interferometer with some modifications. With combination of polarizing beam splitter, quarter wave retarders and a piezoelectric transducer, a controllable linearly polarized light can be produced. The theoretical analysis of the system was performed by Jones matrix algebra and experimentally proved its product using Stokes parameters. The measured values of the degree of polarization and ellipticity of the output beam confirmed the polarization state as linearly polarized light. Preliminary studies of some known samples were carried out to evaluate the application performance of the proposed polarizing interferometer. Samples including quarter and half wave retarders and liquid crystal variable retarder were examined in terms of ellipsometric parameters ψ and Δ. The experimental results were found to be in agreement with corresponding theoretical data.

In terms of the practical achievement, the proposed scheme successfully generate a rotating linearly polarized light beam with

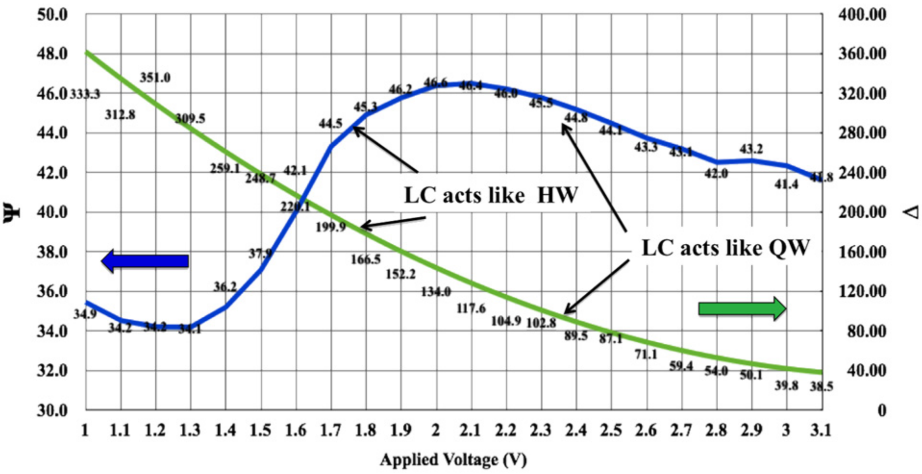

Fig. 6. This illustrates the relationship between the applied voltage and corresponding ellipsometric parameters ψ and Δ.

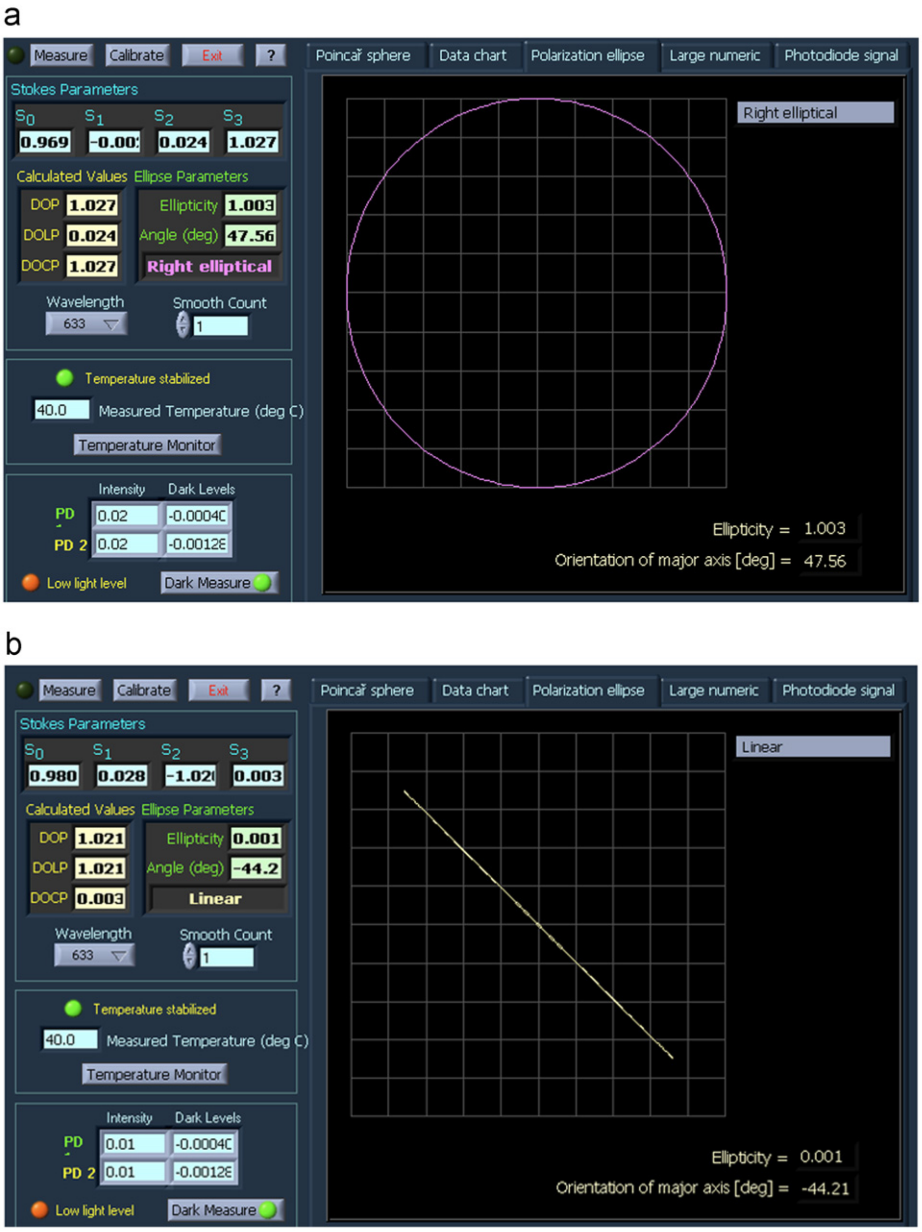

Fig. 7. The state of polarization of the output beam from the liquid crystal with appropriate applied voltage detected by a polarimeter; (a) liquid crystal acting as the quarter wave retarder and (b) liquid crystal acting as the half wave retarder.

a high degree of polarization; i.e. 99.6%. In addition, the proposed interferometer can effectively be applicable for determining the retardance of unknown retarders.

This also should be pointed out that all these successes were possible due to important features from the proposed configuration including (1) inherent reference signal which can be used as a pointer to determine any states of polarization and (2) the output beam, the rotating linearly polarized light, which offers a convenient way to probe a sample characteristics with a signal at different orientations without mechanical movements.

The main future prospect of this study is the implementation of the proposed scheme in optically characterizing substrates and thin films. With minor modifications in the setup arrangement and signal processing, the polarizing Mach–Zehnder interferometer can simply be adapted to perform as an interferometric ellipsometer.

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