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Mixed Control Chart Design Using the Integration of Genetic Algorithm and Monte Carlo Simulation

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Abstract This research proposed a design of mixed control charts to monitoring the process quality based on attribute data together with variable data called a Mixed NE chart. The integration of Genetic Algorithm (GA) and Monte Carlo (MC) simulation are used to simultaneously assess the efficiency of a Mixed NE chart based on three different scenarios of the control limit coefficients $(L_{np}, L_{EWMA\overline{X}-R})$. In this study, MC simulation is used to evaluate the maximum average of run length in case of in-control process ARL_0 while the GA is demanded to optimize the control limit coefficients that obtain the maximum ARL_0. In addition, the efficiency of the Mixed NE chart according to the process shifts are measured using the average of run length in case of out of control process (ARL_1) and the extra quadratic loss(EQL). The results indicate that the Mixed NE chart performed well for small sample size(n), low smoothing constant (λ_w, λ_z) and the optimal design of control limit coefficient is $L^*_{np} > L^*_{EWMA\overline{X}-R}$. Moreover, the optimal design of the Mixed NE chart brings new important perspectives to achieve the most efficiency of control chart.

MSC: 97K80

Keywords: mixed NE chart; genetic algorithm; Monte Carlo simulation; extra quadratic loss function; average of run length

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1. Introduction

Quality control of the industrial process is used to ensure that the products are standard and quality. The widely used techniques of process quality control to monitor and detect the variation occurring in the production process is the statistical control chart (CC). Two types of CCs are normally used to monitor the process in the manufacturing, which introduced by Shewhart [1]. The quality characteristic of items is monitored by the variable CC while the number of defective item is monitored by the attribute CC

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respectively. Many studies claim that Shewhart CC is not effective to monitor a small process shifts. From this problem, Roberts said that the EWMA chart is more efficient to monitor when a small shift in the process has occurred [1]. In addition, many researchers have applied EWMA charts more efficiently, such as, the EWMA chart is used to combine the mean and variance of process into a single CC, called MaxEWMA chart (Chen et al., [2]) The MaxEWMA chart is improved over the Max CC of Chen et al. [3]. Morais and Pacheco [4] presented a combined EWMA(CEWMA) chart to observing the process which measure the efficiency of proposed CC by using the ARLs. Costa and Rahim [5] constructed the single EWMA chart that is applied from the EWMA-SC chart. The result indicates that, their CC have more efficient to monitoring process than $\overline{X} - R$ CCs. Moreover, the study indicates that these single EWMA chart more efficient than the combined of EWMA X chart and EWMA$\ln s^2$ chart in some cases. Khoo et al. [6] proposed a single EWMA chart that combined two Shewhart CCs called EWMA $\overline{X} - R$ chart. The $\overline{X}$ and R statistics are transformed to two EWMA statistics using standard random variables. Then two EWMA statistics are combined into a single plot statistic. Saeed and Kamal [7] compared the performance of six robust scale estimators for EWMA chart for detecting small shifts by calculating expected out-of-control points and expected widths. Raza et al. [8] presented new Shewhart control chart and new EWMA control chart with two phase sampling based on exponential estimator.

Recently, the quality inspection plan is necessary in the industry, many researchers presented a combined CC using advantages of attribute CCs together with variable CCs. Such as, the np chart and $\overline{X}$ chart are combined to observe the mean of a process by Sampaio et al. [9]. The study indicates that these new CC is more efficient than the traditional CC. Aslam et al. [10] designed the new np chart and the $\overline{X}$ chart depend on repetitive sampling. They indicated that the new control charts offer the higher efficiency than the traditional CC. Aslam et al. [11] presented a combined CC to inspect the defective items or monitor a mean of the process using attributes and variables data. Two mixed CCs were designed by Aslam et al. [12]. These CC is used to monitoring the number of defective items. If the decision is uncertain with the attribute CC, the process is changed to monitoring using EWMA statistics or hybrid EWMA(HEWMA) statistics with the variable CC. Ho and Aparisi [13] proposed CCs that are a combination of variable CC together with attribute CC to observe the mean of process, called ATTRIVAR1 and ATTRIVAR2 charts. Ho and Quinino [14] presented the MIX S^2 chart to monitor a variance of the process that is combined between attribute data and variable data.

Based on the literature review, most researchers only use the ARLs to assess the efficiency of the proposed CC when the process shifts. In addition, some studies have considered the effectiveness of proposed CCs using EQL values to analyze the overall efficiency of a CC. Such as, Wu et al. [15] constructed an attribute CC to observe the average of variable, called the np-X chart. The average time to signal (ATS) and EQL are applied to measure the efficiency of modified CC. Ou et al. [16] proposed a comparison of the robustness and effectiveness of nine CCs for monitoring the mean of a variables. The best overall efficiency of the charts are determined by an average of EQL, while the relative overall efficiency of the CCs are measured by Performance Comparison Index (PCL). Haridy et al. [17] applied an attribute CC to monitor when the process shifts in mean and variance. An overall efficiency of these CCs are measured by the ATS and AEQL. They concluded that the proposed CC more effective and less costly than Shewhart CC. The overall efficiency of the $\overline{X} - R$ chart and $\overline{X} - S$ chart are studied by

Haridy et al. [18]. They concluded that the sample size are effects to monitoring the process shifts. The overall efficiency of CC is measured by the AEQL, while the ATS is used to measure for detecting a process shift. Riaz et al. [19] presented a mixed Tukey EWMA-CUSUM chart to enhance process monitoring.They evaluate the efficiency of the mixed CC, under the real lift applications, in term of ARLs while the overall efficiency of these CCs are measured by EQL.

Genetic Algorithm (GA) is one of the most popular method to find the optimal value that provided the best answer of the problem, which is developed from the genetic processes (Holland, [20]). In recently, many researchers indicates that GA can be used to resolve the problem of optimizing in a statistical quality control situation. In example, Charongrattanasakul and Pongpullponsak [21] analyzed the cost of the proposed integrated economic model using the GA approach. Charongrattanasakul and Pongpullponsak [22] applies the fuzzy number to develop their proposed economic model using the GA approach to find optimal variables to minimize cost of model. Moreover, many studies in field of economic statistical design of control chart such as Lina et al. [23], Bashiri et al. [24] and Ahmed et al. [25], applied the GA method to obtain the optimal solution of their proposed economic design model. In studies of control chart design, many researches proposed the optimal design of mixed control chart based on grid search, GA and Monte Carlo simulation to determine the optimal parameters that are corresponding to ARLs. (For example see [9–13]). In addition, some studies designed to hybrid a popular metaheuristic for solving optimization problems. For example, Sombat et al. [26] presented the perspective and experiments of the hybrid algorithm of genetic algorithm and particle swarm optimization to solve the optimization problems. As mentions above, we are interested to apply the integrated optimization method of GA and MC simulation in order to increase the efficiency of mixed CC design.

In this research we modified a mixed CC to observe the process based on attributes data and variables data called Mixed NE chart. Number of defective items are considered based on np chart, but variable data are required based on EWMA $\overline{X} - R$ chart when the decision is in a warning period. The integration of GA and MC simulation were applied to find the optimal value of control limit coefficient of the Mixed NE chart. The ARLs and EQL were calculated to assess the efficiency of the Mixed NE chart according to the process shifts in mean and variance. Numerical examples were used to indicate the efficiency of the Mixed NE chart with three different scenarios. The optimal values of control limit coefficient were considered in the simulation studied that affect the efficiency of the Mixed NE chart.

2. Materials and Methods

In this section, the Mixed NE chart is combined between the attribute CC and the variable CC were presented. This CC is established to monitor the process shifts under the normal distribution. The combined GA and MC simulation were applied to optimize the value of the control limit coefficient as follows.

2.1. EWMA $\overline{X} - R$ Control Chart

Khoo et al. [6] introduced a modified CC called EWMA $\overline{X} - R$ chart. They proposed the methods to combine two Shewhart CCs into the EWMA chart. These CC can be monitors the mean together with variance of process inspection as follow.

Let X_{ij} be a gauge of the process quality characteristic with a normal distributed corresponding to mean Let $\mu_1 = \mu_0 + \delta\sigma_0$ and standard deviation $\sigma_1 = \beta\sigma_0$. If $\delta = 0$ and $\beta = 1$, then the process is in-control (IC) signal. Suppose that i indicate the sample numbers $i = 1, 2, 3, ..., n$ and j indicate observation numbers $j = 1, 2, 3, ..., m$ respectively. Let $\overline{X}_i$ be the ith sample of variable mean and R_i be the ith sample of variable range where $X_{i(m)}$ and $X_{i(1)}$ denotes the largest and smallest gauging in ith sample respectively. As the following equation

$$\overline{X}_i = \frac{\sum_{j=1}^{m} X_{ij}}{m} \tag{1}$$

$$R_i = X_{i(m)} - X_{i(1)}. \tag{2}$$

Supposed that F(.)denote the normal cumulative distribution function (CDF), $N \sim (\mu_0, \sigma_0^2)$. Let $Y_{ij} = F(x_{ij})$ be the jth random observation in ith sample with a uniform distribution $U(0, 1)$. Suppose that R'_i is the ith sample of variable range for $Y_{i(1)}, Y_{i(2)}, ..., Y_{i(m)}$ in ith sample, which $Y_{i(m)}$ and $Y_{i(1)}$ denotes the largest and smallest gauging in ith sample respectively.

From various variables as mentioned above, assuming that U_i and V_i represents random variables that have standard normal distribution of ith sample as shown in the following equation (for further details see Khoo et al., [6]).

$$U_i = \frac{(\overline{X}_i - \mu_0)}{\frac{\sigma_0}{\sqrt{m}}}, i = 1, 2, 3, ... \tag{3}$$

$$V_i = \phi^{-1}G(q'_i), i = 1, 2, 3, ... \tag{4}$$

Assume that $G(q'_i)$ represents the CDF of R'_i and $G(q'_i) = G(q'_i)^{m-1} - (m-1)(q'_i)^m$, $0 < q'_i < 1$, where $q'_i = Y_{i(m)} - Y_{i(1)}$. Let $\phi(Z)$ be the CDF of $Z \sim N(0, 1)$, which $\phi^{-1}(.)$ is the inverse function of $\phi(.)$. The EWMA statistics given by Eq.(5)-(6) respectively.

$$W_i = \lambda_W U_i + (1 - \lambda_W)W_{i-1} \tag{5}$$

$$Z_i = \lambda_Z V_i + (1 - \lambda_Z)Z_{i-1} \tag{6}$$

where $W_0 = Z_0 = 0$ is the starting values, λ_Z and λ_W are the smoothing constant, $0 < \lambda_W, \lambda_Z < 1$, $i = 1, 2, 3, ...$ Suppose W_i and Z_i are combined into M_i as the following Eq.(7)

$$M_i = max\|W_i\|, \|Z_i\|, i = 1, 2, 3, ... \tag{7}$$

The EWMA $\overline{X} - R$ chart is created by considering M_i based on upper control limit because $M_i \geq 0$ as the following equation.

$$UCL_i = E(M_i) + L_{EWMA\overline{X}-R}\sqrt{Var(M_i)}, i = 1, 2, 3, ... \tag{8}$$

$$E(M_i) = \sqrt{\frac{2(\sigma_{W_i}^2) + (\sigma_{Z_i}^2)}{\pi}} \tag{9}$$

$$Var(M_i) = \frac{2}{\pi}[\sigma_{W_i}^2[\arctan(\frac{\sigma_{W_i}}{\sigma_{Z_i}}) - 1] + \sigma_{Z_i}^2[\arctan(\frac{\sigma_{Z_i}}{\sigma_{W_i}}) - 1] + \sigma_{W_i}\sigma_{Z_i}] \tag{10}$$

where $L_{EWMA\overline{X}-R}$ represent control limit coefficient, which is used to determine the width of the EWMA $\overline{X} - R$ chart.

2.2. Design of the Mixed NE Chart

In this section, the Mixed NE chart was constructed by applying the np chart together with the EWMA $\overline{X} - R$ chart. Both CCs are used to inspect the process quality characteristics separately. First, inspection the sample using the gauge to see the defective items under the np chart. If the number of defective item is in the warning period, the inspection process will change to be a variable inspection under the EWMA $\overline{X} - R$ chart. Then, the data set from both CCs will be analyst together as follows.

Step1. Sampling procedure

Select n units of sample with m observation from the data under Normal distribution $N \sim (\mu_0, \sigma_0^2)$.

Step2. Inspection by gauge

Suppose that i indicate the sample numbers $i = 1, 2, 3, ..., n$ and j indicate observation numbers $j = 1, 2, 3, ..., m$ respectively. Assuming X_{ij} represented a measurement of a quality characteristic from a process jth random observation, where $j = 1, 2, 3, ..., m$, in ith sample where $i = 1, 2, 3, ..., n$ respectively. Let X_{ij} be inspected by gauge under the control limit requirements as shown in Eq.(11)

$$UCL_X = \mu_0 + 3\sigma_0 \quad and \quad LCL_X = \mu_0 - 3\sigma_0. \tag{11}$$

If $LCL_X \leq X_{ij} \leq UCL_X$ the unit is declared as accepted; otherwise, it is declared as rejected. Let Y_i denoted number of defective items are rejected by gauge in ith sample.

Step3. Inspection by np chart

In this step, we suppose p_i is the probability that unit classified as rejected in ith sample (from Step 2) then the average proportion of defective items is $\overline{p} = \frac{\sum_{i=1}^{n} p_i}{n}$. Inspect Y_i from Step 2 based on the optimal control limit of np chart as shown in Eq.(12)

$$UCL_{np} = n\overline{p} + L_{np}\sqrt{n\overline{p}(1-\overline{p})} \quad and \quad LCL_{np} = n\overline{p} - L_{np}\sqrt{n\overline{p}(1-\overline{p})} \tag{12}$$

where L_{np} denote the control limit coefficient which it is used to control the width of the np chart. Let w be the limit to remind the process is in the warning period.

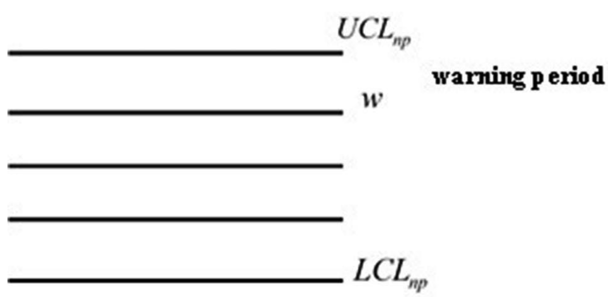

Figure 1. Procedure of np chart

From Figure 1, notify that the process is out of control (OCC) if $Y_i > UCL_{np}$. Otherwise, if $w > Y_i$ the process notify is IC signal. In addition, if $w \leq Y_i \leq UCL_{np}$ then the next sample in this process will be monitored using EWMA $\overline{X} - R$ (go to next step).

Step4. Inspection by EWMA $\overline{X} - R$

Use the same unit of sample from 2.2.1 to compute the $\overline{X}_i$ and R_i that are computed in Eq.(1)-(2). Then, the value of U_i, V_i, W_i, Z_i and $M_i, i = 1, 2, 3, ..., n$ are computed in Eq.(3)-(7). Next, M_i is inspected based on the optimal control limit of EWMA $\overline{X} - R$ chart. Notify that is the OCC signal if $M_i > UCL_i$. Otherwise, if $M_i \leq UCL_i$ notify that is IC signal as shown in Figure 2.

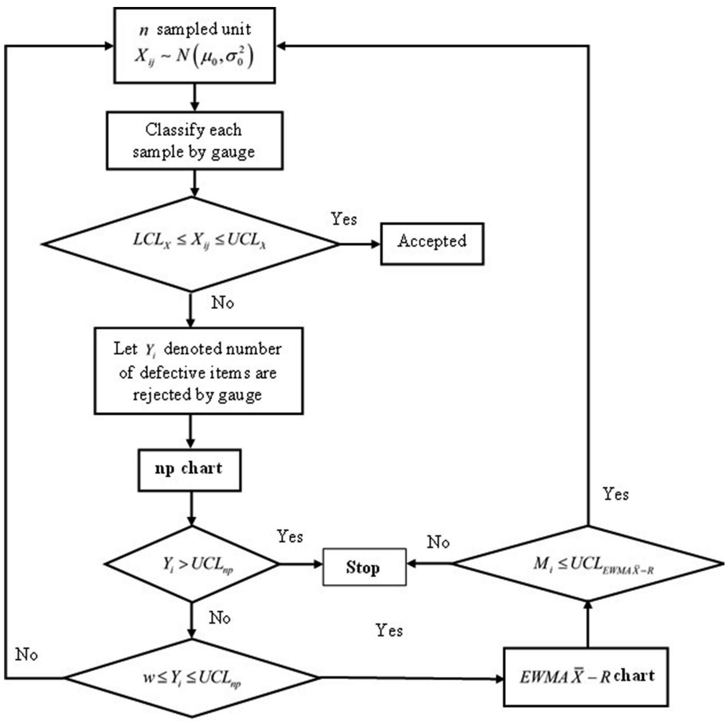

Figure 2. Inspection procedure for the Mixed NE chart

2.3. Efficiency Analysis of Control Chart

The efficiency of CCs can be measured by using various methods. In this research, there are two classifications of these measures. The ARLs are used to measures the process for specific shifts while the EQL is used to measures the overall process shifts as follows.

Step1. Average of Run Length

There are two type of ARL, which are denoted by ARL_0 and ARL_1. The ARL_s are used to evaluate the efficiency. Under the conditions of the IC signal, the mean number of samples that are within the control limit before the OCC signal denoted by ARL_0 , while the mean number of samples when the process shift to the OCC signal is denoted by ARL_1 as follows.

The average of run length of process in case of IC signal is $ARL_0 = \frac{1}{\alpha}$, suppose α denotes the probability of summarize that process is in the OCC signal, but in actually, the process is IC signal as follow in Eq.(13).

$$\alpha = P[(Y_i > UCL_{np}), \mu{=}\mu_0] + 1 - P[(M_i \leq UCL_{EWMA\overline{X}-R}|w \leq Y_i \leq UCL_{np}), \mu{=}\mu_0] \tag{13}$$

The average of run length in case of OCC process is $ARL_1 = \frac{1}{1-\beta}$, suppose β denote the probability of summarize that process is the IC signal, but in actually, the process is OCC signal as follow in Eq.(14).

$$1-\beta{=}P[(Y_i{>}UCL_{np}), \mu{=}\mu_1] + 1 - P[(M_i{\leq}UCL_{EWMA\overline{X}-R}|w \leq Y_i \leq UCL_{np}), \mu{=}\mu_1] \tag{14}$$

Step2. Extra Quadratic Loss

In situation of the efficient of CC, the shift size and the quality impact of CC are related that EQL is calculated to analyses these relationship. The overall efficiency of a CC is measured in a range of shifts $(0 < \delta < \delta_{max})$. The smaller EQL, the better overall efficiency of the CC. The EQL is calculated as follow Eq.(15).

$$EQL(\delta) = \int_{\delta_{min}}^{\delta_{max}} w(\delta)ARL(\delta)f(\delta)d\delta \tag{15}$$

where δ_{min} and δ_{max} are the lower bound and the upper bound of range in case the process shifts respectively. Suppose δ_{min} is set as zero for simple to calculations. Let $w(\delta)$ be the weight function of δ with $w(\delta) = \delta^2$. Supposed that δ occur with equal probability, which a density function of a uniform distribution is $f(\delta) = \frac{1}{\delta_{max}}$. In actual calculations, the integration in Eq.(15) can be estimated by the summation. Consequently, Eq.(15) can be further simplified as follows Eq.(16) (for more details see [16]).

$$EQL(\delta) = \frac{1}{\delta_{max}} \sum_{\delta=0}^{\delta_{max}} ARL(\delta). \tag{16}$$

In addition, the process shifts in mean and variance $(0 < \delta < \delta_{max}, 1 < \beta < \beta_{max})$, EQL is adapt to observe the efficiency of a CC which the weight function of δ and β given as $(\delta^2 + \beta^2 - 1)$. Then, the EQL for both process shifts can be calculated in Eq.(17)

$$EQL(\delta, \beta) = \int_0^{\delta_{max}} \int_0^{\beta_{max}} (\delta^2 + \beta^2 - 1)ARL(\delta, \beta)f(\delta)f(\beta)d\delta d\beta. \tag{17}$$

Suppose that δ and β occur with equal probability, therefore, a density function of a uniform distribution are $f(\delta) = \frac{1}{\delta_{max}}$ and $f(\beta) = \frac{1}{\beta_{max}-1}$ respectively. Similarly as Eq.(16) the integration can be approximated by the summation. Then, the expression of EQL given in Eq.(18)

$$EQL(\delta, \beta) = \frac{1}{\delta_{max}(\beta_{max} - 1)} \sum_{\delta=0}^{\delta_{max}} \sum_{\beta=1}^{\beta_{max}} (\delta^2 + \beta^2 - 1)ARL(\delta, \beta). \tag{18}$$

2.4. Optimization Methods Using Genetic Algorithm

Genetic Algorithm (GA) is one of the most popular method to find the optimal value that provided the best answer of the problem, which is developed from the genetic processes [20]. The data will be considered in the form of encoding called chromosomes, which is transmitted from the parent chromosome to their child chromosome. The objective function is defined in accordance with the problem by considering the fitness value of chromosome for problems that need to be calculated. To do this, genetic operators are taken with the initial chromosomes until the new chromosome is the most suitable for the problem. The procedure for the GA is briefly described as illustrating in Figure 3.

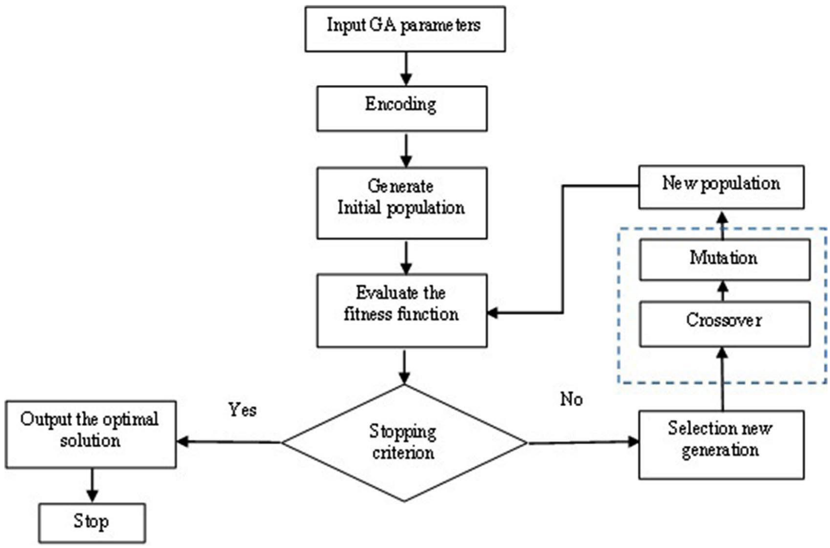

Figure 3. The solution procedure for the GA

1) **Chromosome encoding** is the first step for solving the problem using GA by starting to convert the data type of the problem to chromosomes.

2) **Initial population** is randomly selected to create a prototype population that is the starting point of the process.The fifty initial solutions that satisfy the constraint condition are randomly produced. The constraint condition for control chart coefficient of each control chart is set as $2 < L_{np} < 3$, $2 < L_{EWMA\overline{X}-R} < 3$.

3) **Fitness function** is a method of assessing opportuneness to provide points for answers to problems. The optimal chromosome is used to transferring genes to create a new generation of chromosomes. The fitness function for our studies is the maximum ARL_0.

4) **Selection** is a method for finding a survival of organisms by selecting them as parent chromosome. The survival chromosomes are selected for the next generation according to the better fitness of chromosomes. The most satisfy selection of chromosomes is done for reproduction.

5) **Crossover** is the method in order to make chromosomes more changeable by mixing two chromosomes together then the new chromosome has occurred which two of survivors (from the 50 solutions) are selected randomly as the parents used for crossover operations

to produce new chromosomes for the next generation. In this study the crossover rate is 0.9.

6) **Mutation** is a random method to choose a solution, which each value within the solution will be changed randomly at a mutation rate between 0.01 to 0.1. In this study the mutation rate is 0.1.

7) **Stopping condition** when a pre-selected number of generations are achieved. After that, a few preliminary tests are performed to obtain a reasonable number. In this study, we use fifty generations as stopping criteria.

3. Numerical Example

In this section, the efficiency analysis of the mixed CC are considered. In literature reviews, many researches used the only Monte Carlo simulation or optimization method to find the control limit coefficients L_{np} and $L_{EWMA\overline{X}-R}$ by fix the value of ARL_0. This research proposes a method to calculate the optimal value of L^*_{np} and $L^*_{EWMA\overline{X}-R}$ that maximize ARL_0 for various values of n with the optimization model using the integration of GA and MC simulation. In GA method, we supposed that the fixed value of initial population is 50 generations. The crossover rate is 0.90 and the mutation rate is 0.01, while the MC simulation has the repetition 10,000 times respectively.

In recently, many studies such as [10–14] proposed a mixed control chart, but the value of control chart coefficient between two CCs are not compared. From this situation, the aim of this research proposed three different scenarios between L_{np} and $L_{EWMA\overline{X}-R}$ that affects to the efficiency of Mixed NE chart as follows Eq.(19)-(20).

$$\text{Maximize} \quad ARL_0 \tag{19}$$

$$\text{Subject to} \quad 2 < L_{np}, L_{EWMA\overline{X}-R} < 3. \tag{20}$$

Scenario 1 (S1): $L_{np} > L_{EWMA\overline{X}-R}$
Scenario 2 (S2): $L_{np} < L_{EWMA\overline{X}-R}$
Scenario 3 (S3): $L_{np} = L_{EWMA\overline{X}-R}$.

The procedure to analyst the efficiency of the Mixed NE chart shown as follows.

Phase I: A random variable X_{ij} is generated with normal distribution $N(\mu_0 = 500, \sigma_0^2 = 100)$ (The data are given in [6]) with $m = 100$ observation and select $n = 4, 5$ and 6 units of sample respectively. The integration of GA and MC simulation are used to generate the optimal L^*_{np} and $L^*_{EWMA\overline{X}-R}$ for each scenario to maximize ARL_0 at $\delta = 0$ and $\beta = 1$ as shown in Figure 4.

Phase II : ARL_1 and EQL are determined to assess the efficiency of the Mixed NE chart when the process shifts using the optimal L^*_{np} and $L^*_{EWMA\overline{X}-R}$ in phase I. ARL_1 and EQL are considered based on three different scenarios which is estimated by MC simulation with $T_{max} = 10,000$ repetitions. The fixed value of mean shift are $\delta = 0.25, 0.5, 1, 1.25, 1.5$ and variance shift are $\beta = 1, 1.5, 2, 2.5$ respectively.

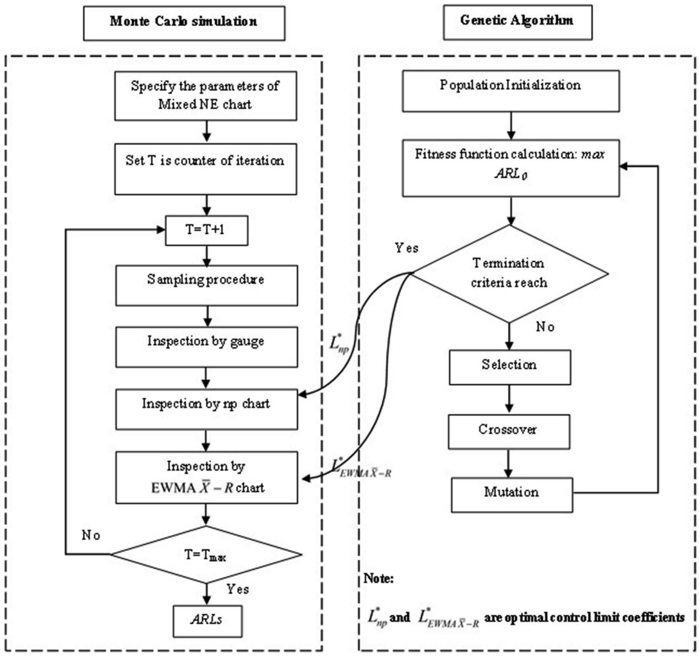

Figure 4. Algorithm of the integration of GA and MC simulation

4. Results

In this situation, the ARL_s and EQL are indicated the advantage of the Mixed NE chart. In case of IC process, GA is applied to obtain the optimal L^*_{np} and $L^*_{EWMA\overline{X}-R}$ corresponding to maximum ARL_0 with three different scenarios for some specific sample size($n = 4, 5, 6$), warning limit of the np CC ($w = 2$) and the fixed value of the smoothing constant are $\lambda_w = \lambda_z = 0.2, 0.4, 0.6$ and 0.8 respectively.

From Table 1, the optimal L^*_{np} and $L^*_{EWMA\overline{X}-R}$ corresponding to maximum ARL_0 are optimized by the integration of GA and MC simulation. Considering in each value of λ_W and λ_Z, it is indicated that the Mixed NE chart has the higher ARL_0 when the smoothing constant $0 < \lambda_W, \lambda_Z < 1$ approach to one. If considering in each sample size, results shown that the Mixed NE chart is the higher ARL_0 when a sample size is decreased respectively. Next, the OCC efficiency of the Mixed NE chart can be investigated. To do this, ARL_1 and EQL are used to assess the efficiency of the proposed mixed CCs according to process shift in mean and variance by Monte Carlo simulation with 10,000 repetitions. Considering the overall values of ARL_1 and EQL of the Mixed NE chart in cause of the process shifts in Tables 2-3. The results indicate that, considering in each value of λ_W and λ_Z, ARL_1 is decreasing when δ or β are increasing. Moreover, the Mixed NE chart has the lowest $EQL$when the smoothing constant approach to zero. If considering in each sample size, results shown that the Mixed NE chart is the lower ARL_1 when a sample size is increased respectively.

From Table 2, the Mixed NE chart that corresponding to three different scenarios are investigated with process shift in mean (mean increases and variance remain constant). The optimal design for $n = 4$ are resulted, S3 provides the best efficiency (lowest ARL_1

and EQL)of the Mixed NE chart when $\lambda_W = \lambda_Z = 0.2$, S2 provides the best efficiency when $\lambda_W = \lambda_Z = 0.4$ and 0.6 while S1 provides the best efficiency when $\lambda_W = \lambda_Z = 0.8$ respectively. The optimal design for $n = 5$ are resulted, S3 provides the best efficiency when $\lambda_W = \lambda_Z = 0.2$ and 0.6 , S1 provides the best efficiency when $\lambda_W = \lambda_Z = 0.4$ while S2 provides the best efficiency when $\lambda_W = \lambda_Z = 0.8$ respectively. The optimal design for $n = 6$ are resulted, S2 provides the best efficiency when $\lambda_W = \lambda_Z = 0.2$, S1 and S2 provides the best efficiency when $\lambda_W = \lambda_Z = 0.4$, S2 provides the best efficiency when $\lambda_W = \lambda_Z = 0.6$ and S3 provides the best efficiency when $\lambda_W = \lambda_Z = 0.8$ respectively.

From Table 3, the efficiency of the Mixed NE chart that corresponding to 3 different scenarios are investigated with process shift in variance (variance increases and mean remain constant). The optimal design for $n = 4$ are observed, S1 provides the best efficiency of the CC when $\lambda_W = \lambda_Z = 0.2, 0.6$ and 0.8 while S3 provides the best efficiency when $\lambda_W = \lambda_Z = 0.4$ respectively. The optimal design for $n = 5$ are observed, S2 provides the best efficiency when $\lambda_W = \lambda_Z = 0.2$, S1 provides the best efficiency when $\lambda_W = \lambda_Z = 0.4$, while S3 provides the best efficiency when $\lambda_W = \lambda_Z = 0.6$ and 0.8 respectively. The optimal design $n = 6$ are observed, S1 provides the best efficiency when $\lambda_W = \lambda_Z = 0.2$ and 0.4, S1 provides the best efficiency when $\lambda_W = \lambda_Z = 0.4$, S2 provides the best efficiency when $\lambda_W = \lambda_Z = 0.6$ and S3 provides the best efficiency when $\lambda_W = \lambda_Z = 0.6$ and 0.8 respectively. The results in Tables 2-3 indicated that the value $\lambda_W = \lambda_Z = 0.2$ provides the Mixed NE chart has the best efficiency. In addition, the optimal scenario of L^*_{np} and $L^*_{EWMA\overline{X}-R}$ with respect to $n = 4, 5$ and 6 from Tables 2-3 can be summarized as follows in Table 4. In addition, the efficiency of the Mixed NE chart that corresponding to 3 different scenarios are investigated with process sifts in mean and variance when $\lambda_W = \lambda_Z = 0.2$, $n = 4, 5$ and 6 as follows in Table 5. The results shown that S1 provides the best efficiency of the Mixed NE chart.

TABLE 1. The optimal L^*_{np} and $L^*_{EWMA\overline{X}-R}$ corresponding to maximum ARL_0 at $\delta = 0$ and $\beta = 1$ [$((L^*_{np}, L^*_{EWMA\overline{X}-R}))$ = (A,B)]

$n = 4$						
	S1		S2		S3	
$\lambda_W = \lambda_Z$	(A,B)	ARL_0	(A,B)	ARL_0	(A,B)	ARL_0
0.2	(2.55,2.23)	348.36	(2.42,2.77)	349.43	(2.08,2.08)	348.70
0.4	(2.79,2.47)	349.14	(2.03,2.20)	349.49	(2.13,2.13)	349.17
0.6	(2.35,2.24)	348.59	(2.10,2.20)	349.49	(2.38,2.38)	349.43
0.8	(2.59,2.01)	350.55	(2.21,2.68)	349.63	(2.71,2.71)	351.15
$n = 5$						
	S1		S2		S3	
$\lambda_W = \lambda_Z$	(A,B)	ARL_0	(A,B)	ARL_0	(A,B)	ARL_0
0.2	(2.58,2.20)	180.90	(2.18,2.52)	180.48	(2.29,2.29)	184.04
0.4	(2.50,2.20)	180.99	(2.37,2.78)	181.55	(2.37,2.37)	181.27
0.6	(2.55,2.26)	181.43	(2.26,2.51)	181.93	(2.39,2.39)	182.71
0.8	(2.67,2.23)	193.60	(2.21,2.61)	194.39	(2.52,2.52)	195.74
$n = 6$						
	S1		S2		S3	
$\lambda_W = \lambda_Z$	(A,B)	ARL_0	(A,B)	ARL_0	(A,B)	ARL_0
0.2	(2.74,2.40)	41.09	(2.33,2.53)	42.29	(2.57,2.57)	42.53
0.4	(2.70,2.47)	44.58	(2.36,2.63)	45.57	(2.81,2.81)	44.79
0.6	(2.75,2.43)	53.34	(2.24,2.73)	52.70	(2.78,2.78)	53.36
0.8	(2.82,2.41)	88.84	(2.25,2.55)	88.50	(2.56,2.56)	88.20

TABLE 2. The values of ARL_1 and EQL of the Mixed NE chart in cause of the process shift in mean($\mu = \mu_0 + \delta\sigma_0$)where $n = 4, 5$ and 6

$n=4$									
			δ						
	$\lambda_W = \lambda_Z$	$(L^*_{np}, L^*_{EWMA\overline{X}-R})$	0.25	0.5	0.75	1	1.25	1.5	EQL
S1	0.2	(2.55,2.23)	332.14	299.22	205.02	88.61	27.65	6.70	47.97
	0.4	(2.79,2.47)	332.43	297.60	206.91	90.45	32.06	14.03	48.67
	0.6	(2.35,2.24)	328.22	296.76	205.73	94.43	40.51	27.46	49.66
	0.8	(2.59,2.01)	333.95	297.09	213.21	113.70	76.70	73.04	55.38
S2	0.2	(2.42,2.77)	328.92	297.76	208.33	90.63	27.06	6.96	47.98
	0.4	(2.03,2.20)	331.41	294.76	204.20	90.45	32.06	14.03	48.35
	0.6	(2.10,2.20)	330.52	291.91	210.38	92.30	14.03	27.35	49.59
	0.8	(2.21,2.68)	333.95	297.98	215.04	115.20	48.35	73.07	55.60
S3	0.2	(2.08,2.08)	331.14	296.72	203.91	88.93	28.49	8.25	47.87
	0.4	(2.13,2.13)	332.24	296.20	209.49	91.36	31.42	14.18	48.74
	0.6	(2.38,2.38)	329.52	297.12	206.50	95.28	39.70	27.41	49.78
	0.8	(2.71.2.71)	330.58	299.62	217.13	115.33	76.85	73.05	55.63
$n=5$									
			δ						
	$\lambda_W = \lambda_Z$	$(L^*_{np}, L^*_{EWMA\overline{X}-R})$	0.25	0.5	0.75	1	1.25	1.5	EQL
S1	0.2	(2.58,2.20)	142.73	71.61	26.12	7.61	2.03	1.06	12.56
	0.4	(2.50,2.20)	139.63	73.62	29.23	13.17	10.22	10.00	13.79
	0.6	(2.55,2.26)	143.47	78.83	37.47	27.08	26.01	26.00	16.94
	0.8	(2.67,2.23)	158.07	102.36	76.06	73.03	73.00	73.00	27.78
S2	0.2	(2.18,2.52)	140.09	72.82	25.08	6.74	1.94	1.07	12.39
	0.4	(2.37,2.78)	143.65	74.42	29.45	12.93	10.23	10.01	14.03
	0.6	(2.26,2.51)	146.96	76.76	37.82	27.04	26.01	26.00	17.03
	0.8	(2.21,2.61)	155.94	102.2	75.97	73.03	73.00	73.00	27.66
S3	0.2	(2.29,2.29)	139.3	73.1	24.66	7.25	1.87	1.08	12.36
	0.4	(2.37,2.37)	141.4	74.66	28.37	13.17	10.21	10.01	13.89
	0.6	(2.39,2.39)	142.58	79.18	37.16	27.02	26.02	26.00	16.90
	0.8	(2.52,2.52)	161.1	102.9	76.03	73.04	73.02	73.00	27.96
$n=6$									
			δ						
	$\lambda_W = \lambda_Z$	$(L^*_{np}, L^*_{EWMA\overline{X}-R})$	0.25	0.5	0.75	1	1.25	1.5	EQL
S1	0.2	(2.74,2.40)	24.34	10.57	3.42	1.26	1.01	1.00	2.08
	0.4	(2.70,2.47)	27.05	15.97	10.77	10.03	10.00	10.00	4.19
	0.6	(2.75,2.43)	36.25	28.48	26.16	26.00	26.00	26.00	8.44
	0.8	(2.82,2.41)	75.55	73.14	73.00	73.00	73.00	73.00	22.03
S2	0.2	(2.33,2.53)	23.37	10.64	3.39	1.27	1.01	1.00	2.03
	0.4	(2.36,2.63)	27.14	15.82	10.82	10.04	10.00	10.00	4.19
	0.6	(2.24,2.73)	36.68	28.57	26.14	26.00	26.00	26.00	8.47
	0.8	(2.25,2.55)	75.73	73.24	73.00	73.00	73.00	73.00	22.05
S3	0.2	(2.57,2.57)	23.49	10.72	3.40	1.23	1.01	1.00	2.04
	0.4	(2.81,2.81)	27.72	15.88	10.86	10.04	10.00	10.00	4.22
	0.6	(2.78,2.78)	36.9	28.48	26.15	26.00	26.00	26.00	8.48
	0.8	(2.56,2.56)	75.13	73.16	73.00	73.00	73.00	73.00	22.01

TABLE 3. The values of ARL_1 and EQL of the Mixed NE chart in cause of the process shift in variance($\sigma = \beta\sigma_0$)where $n = 4, 5$ and 6

$n = 4$									
			δ						
	$\lambda_W = \lambda_Z$	$(L^*_{np}, L^*_{EWMA\overline{X}-R})$	0.25	0.5	0.75	1	1.25	1.5	EQL
S1	0.2	(2.55,2.23)	219.97	87.09	33.62	14.96	474.19	1.00	2.08
	0.4	(2.79,2.47)	221.01	86.85	36.53	19.79	485.58	10.00	4.19
	0.6	(2.35,2.24)	222.26	92.42	43.4	30.53	518.14	26.00	8.44
	0.8	(2.59,2.01)	224.32	111.86	78.71	73.52	651.21	73.00	22.03
S2	0.2	(2.42,2.77)	219.87	88.62	33.45	15.10	476.05	1.00	2.03
	0.4	(2.03,2.20)	221.01	88.75	36.08	20.04	487.84	10.00	4.19
	0.6	(2.10,2.20)	225.04	92.73	43.95	30.73	523.27	26.00	8.47
	0.8	(2.21,2.68)	229.11	112.01	78.92	73.64	658.23	73.00	22.05
S3	0.2	(2.08,2.08)	222.3	86.25	33.13	15.29	475.96	1.00	2.04
	0.4	(2.13,2.13)	218.58	85.83	36.2	19.41	480.03	10.00	4.22
	0.6	(2.38,2.38)	222.3	92.55	43.85	31.12	519.76	26.00	8.48
	0.8	(2.71.2.71)	228.54	111.86	78.72	73.59	656.95	73.00	22.01
$n = 5$									
			δ						
	$\lambda_W = \lambda_Z$	$(L^*_{np}, L^*_{EWMA\overline{X}-R})$	0.25	0.5	0.75	1	1.25	1.5	EQL
S1	0.2	(2.58,2.20)	31.69	7.75	2.65	1.45	58.06	1.00	2.08
	0.4	(2.50,2.20)	33.39	13.47	10.51	10.06	89.89	10.00	4.19
	0.6	(2.55,2.26)	42.38	27.21	26.06	26.00	162.19	26.00	8.44
	0.8	(2.67,2.23)	77.97	73.04	73.00	73.00	396.02	73.00	22.03
S2	0.2	(2.18,2.52)	30.74	7.52	2.60	1.42	56.37	1.00	2.03
	0.4	(2.37,2.78)	34.36	13.38	10.51	10.05	91.06	10.00	4.19
	0.6	(2.26,2.51)	41.49	27.25	26.05	26.00	161.05	26.00	8.47
	0.8	(2.21,2.61)	77.95	73.00	73.00	73.00	395.94	73.00	22.05
S3	0.2	(2.29,2.29)	31.45	7.37	2.61	1.44	57.16	1.00	2.04
	0.4	(2.37,2.37)	34.09	13.39	10.51	10.08	90.75	10.00	4.22
	0.6	(2.39,2.39)	41.31	26.99	26.06	26.00	160.48	26.00	8.48
	0.8	(2.52,2.52)	77.89	73.04	73.00	73.00	395.91	73.00	22.01
$n = 6$									
			δ						
	$\lambda_W = \lambda_Z$	$(L^*_{np}, L^*_{EWMA\overline{X}-R})$	0.25	0.5	0.75	1	1.25	1.5	EQL
S1	0.2	(2.74,2.40)	4.22	1.34	1.04	1.00	10.14	1.00	2.08
	0.4	(2.70,2.47)	10.00	10.05	10	10.00	53.41	10.00	4.19
	0.6	(2.75,2.43)	26.25	26.00	26.00	26.00	139.00	26.00	8.44
	0.8	(2.82,2.41)	73.00	73.00	73.00	73.00	389.34	73.00	22.03
S2	0.2	(2.33,2.53)	4.24	1.34	1.05	1.00	10.17	1.00	2.03
	0.4	(2.36,2.63)	11.34	10.05	10.00	10.00	55.19	10.00	4.19
	0.6	(2.24,2.73)	26.26	26.01	26.00	26.00	139.03	26.00	8.47
	0.8	(2.25,2.55)	73.01	73.00	73.00	73.00	389.35	73.00	22.05
S3	0.2	(2.57,2.57)	4.23	1.35	1.03	1.01	10.15	1.00	2.04
	0.4	(2.81,2.81)	11.32	10.07	10.00	10.00	55.19	10.00	4.22
	0.6	(2.78,2.78)	26.24	26.00	26.00	26.00	138.98	26.00	8.48
	0.8	(2.56,2.56)	73.00	73.00	73.00	73.00	389.33	73.00	22.01

Table 4. Summaries of the optimal scenarios of L^*_{np} and $L^*_{EWMA\overline{X}-R}$ with respect to $n = 4, 5$ and 6

		$\lambda_W = \lambda_Z$			
Type of process shift	Sample size (n)	0.2	0.4	0.6	0.8
mean shifts	4	S 3	S 2	S 2	S 1
	5	S 3	S 1	S 3	S 2
	6	S 2	S1 and S2	S2	S3
variance shifts	4	S 1	S 3	S 1	S 1
	5	S 2	S 1	S 3	S 3
	6	S 1	S 1	S 3	S 3

Table 5. The values of ARL_1 and EQL of the Mixed NE chart in cause of the process shift in mean and variance where $n = 4, 5$ and 6

$n=4, \lambda_W=\lambda_Z=0.2, (L^*_{np}, L^*_{EWMA\overline{X}-R})$=(2.55,2.23)												
	S1				S2				S3			
	β				β				β			
δ	1.25	1.5	1.75	2	1.25	1.5	1.75	2	1.25	1.5	1.75	2
0.25	194.61	71.73	27.34	12.94	195.04	71.73	27.34	12.94	197.34	70.28	28.65	12.50
0.50	124.27	41.06	15.85	7.42	125.16	41.06	15.85	7.42	123.92	40.22	16.02	7.34
0.75	54.06	17.51	7.51	3.59	54.84	17.51	7.51	3.59	56.76	17.74	7.38	3.54
1	18.88	6.49	2.85	1.60	18.88	6.49	2.85	1.60	18.76	6.41	2.93	1.72
1.25	5.92	2.14	1.35	1.12	5.86	2.14	1.35	1.12	5.80	22.00	1.30	1.11
1.50	1.93	1.17	1.02	1.01	1.99	1.17	1.02	1.01	1.99	1.18	1.04	1.01
EQL	512.62				512.78				514.70			
$n=5, \lambda_W=\lambda_Z=0.2, (L^*_{np}, L^*_{EWMA\overline{X}-R})$=(2.58,2.20)												
	S1				S2				S3			
	β				β				β			
δ	1.25	1.5	1.75	2	1.25	1.5	1.75	2	1.25	1.5	1.75	2
0.25	24.21	6.24	2.18	1.29	24.51	6.16	2.23	1.31	23.66	6.24	2.18	1.29
0.50	12.82	3.13	1.47	1.10	11.46	3.14	1.45	1.11	12.82	3.13	1.47	1.10
0.75	4.62	1.51	1.07	1.01	4.37	1.53	1.09	1.01	4.62	1.51	1.07	1.01
1	1.59	1.05	1.01	1.00	1.60	1.06	1.00	1.00	1.59	1.05	1.01	1.00
1.25	1.03	1.00	1.00	1.00	1.04	1.00	1.00	1.00	1.04	1.00	1.00	1.00
1.50	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
EQL	72.92				73.46				73.91			
$n=6, \lambda_W=\lambda_Z=0.2, (L^*_{np}, L^*_{EWMA\overline{X}-R})$=(2.74,2.40)												
	S1				S2				S3			
	β				β				β			
δ	1.25	1.5	1.75	2	1.25	1.5	1.75	2	1.25	1.5	1.75	2
0.25	3.31	1.24	1.02	1.00	3.25	1.25	1.01	1.00	3.38	1.22	1.02	1.01
0.50	1.78	1.06	1.00	1.00	1.79	1.06	1.00	1.00	1.78	1.06	1.00	1.00
0.75	1.12	1.01	1.00	1.00	1.14	1.00	1.00	1.00	1.11	1.00	1.00	1.00
1	1.00	1.00	1.00	1.00	1.01	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.25	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.50	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
EQL	44.45				44.46				44.47			

5. Conclusion and Discussion

In this research, the Mixed NE chart is designed to observe the process quality based on both of attributes data and variables data. The np chart together with the EWMA $\overline{X}-R$ chart are applied with process inspection. The np chart is used to inspect attributes data while the EWMA $\overline{X}-R$ chart is used to inspect variables data. Both CCs are used to inspect the quality in the process separately. In situation of IC process, the optimal value of L^*_{np} and $L^*_{EWMA\overline{X}-R}$ in three different scenarios are obtained by the integration of GA and MC simulation. The Mixed NE chart with the smallest sample size ($n = 4$) provide the highest ARL_0 when λ_W and λ_Z approach to one. On the other side, ARL_1 and EQL are used to assess the efficiency of the Mixed NE chart according to the mean and variance of process shifts. In situation that the process shifts, the results indicated that the Mixed NE chart is suitable for the smallest sample size and lowest smoothing constant respectively. At the smallest sample size, S3 ($L^*_{np} = L^*_{EWMA\overline{X}-R}$) is optimal design for the Mixed NE chart when the process shifts in mean, while S1 ($L^*_{np} > L^*_{EWMA\overline{X}-R}$) is optimal design when the process shifts in variance. In addition, both of mean and variance shifts, the results shown that the optimal designed is S1 ($L^*_{np} > L^*_{EWMA\overline{X}-R}$) respectively.

In conclusion, the results of the study indicated that the optimal value of L^*_{np} and $L^*_{EWMA\overline{X}-R}$ will affect to the efficiency of the Mixed NE chart. We found that the best scenario of control chart coefficient is $L^*_{np} > L^*_{EWMA\overline{X}-R}$ which it provided more effective to detection of process shifts. Based on this study, we also suggest that manufacturer should select the smallest sample size, the lowest smoothing constant and the optimal control chart coefficient is S1 to achieve the desired efficiency.

In future research, Mixed NE chart will be consider when variable data based on other non-symmetric distributions. Moreover, the other variable control chart is used to inspect variables data such as MaxEWMA chart, HEWMA chart and HEWMA-CUSUM chart. Then all of proposed control chart will compare to analysis the efficiency between the control charts to get the most effective of mixed control chart design.

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